

Market Counterfactuals with Nonparametric Supply: An ML/AI Approach

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July 7, 2025 — Alpine IO Symposium

Motivation

- The outcomes of government policy are shaped by the response of markets
- Thus, we seek answers to a wide range of policy questions from empirical models of demand and supply, used to perform market counterfactuals
 - E.g., effect of taxes and subsidies, product regulations, competition policy, and more
- Credible empirical models of markets require balance between data and structure
- Key achievement of IO approach: enable flexible estimation of demand, rich subst. patterns
 - Otherwise, ans. to counterfactual questions driven by assumption/parametric restrictions
- What about supply?

Motivation

- Market counterfactuals crucially depend on the **supply** (i.e., conduct and cost) specification
- Examples:
 - Price response to selective tax/tariff depends on strategic complements/substitutes
 - Economies of scale/scope matter in response to industry consolidation/mergers
- Standard approach imposes strong assumptions on supply
 - E.g., Bertrand-Nash oligopoly with constant marginal cost
- Can we provide useful market counterfactuals without restrictive assumptions on supply?

What We Do I

- Consider a **nonparametric model of supply**
- Supply fct. that captures markup + costs, depends on endogenous prices and quantities
- Reflects notion that prices are set in market equilibrium, but no structure on conduct or cost
- Show identification with appropriate **supply instruments**
 - Can leverage both variation in (own and rival) demand shifters, and rival cost shifters

What We Do II

- Estimate model with ML/AI: deep learning + objective function with instruments
- Adapt **Variational Method of Moments (VMM)** (Bennett and Kallus, 2023)
- Better performance with high-dimensional data than standard nonpar-IV (e.g., series)
- We develop an inference procedure to quantify uncertainty in prediction

What We Do III

- Simulations show that nonparametric model is practical with moderate sample sizes/variation, outperforms misspecified models
- Simulations across many counterfactuals showcase performance
- Application: mergers in airline markets
- Portable method, computationally manageable

- We build on existing **nonparametric** approaches to markets for differentiated products
 - Nonparametric identification: Berry and Haile (2014)
 - Nonparametric approaches to demand: Compiani (2022), Tebaldi, Torgovistky, and Yang (2023), Brand and Smith (2025), ...
 - Flexible approaches to supply: Gandhi and Houde (2020), Otsu and Pesendorfer (2024)
- Complementary to **testing**/parametric estimation approaches
 - Modern testing approaches (e.g., Backus, Conlon, and Sinkinson 2021, Duarte et al. 2023) also let data shape supply by selecting conduct model within menu
 - Trade-off: flexibility vs. data/variation requirements
- Part of broader trend of using **ML/AI** to enhance structural modeling
 - E.g., Kaji, Manresa, and Pouliot (2023)
 - We use Bennet and Kallus (2023) for ML/AI approach to nonparametric IV

Roadmap

Market Equilibria and Counterfactuals

Nonparametric Model of Supply

Estimation and Inference

Monte Carlo Simulations

Empirical Application

Conclusion

A Model of Market Equilibrium

- We observe data on a set of differentiated products \mathcal{J} across \mathcal{T} markets:
 - Consumers and firms' behavior results in outcomes p_{jt} (prices) and s_{jt} (market shares)
 - Exogenous observables include characteristics x_{jt} , cost shifters w_{jt}
 - Exogenous unobservables are unobserved quality ξ_{jt} and unobserved cost shifter ω_{jt}
 - Useful transformations of endogenous variables include demand derivatives D_t and quantities q_t
- Market equilibrium is determined by demand and supply:

$$s_t = \underbrace{\delta(p_t, x_t, \xi_t)}_{\text{Demand}}, \quad p_t = \underbrace{\Delta(p_t, s_t, D_t; \cdot)}_{\text{Markups}} + \underbrace{c(q_t, w_t, \omega_t)}_{\text{Marginal costs}}$$

- Markups can depend on other exogenous variables, e.g., ownership matrix \mathcal{H}_t
- Allow for conduct and cost to depend on firm identity, denote Δ_j, c_j scalar valued functions
- General static setting, can be extended to other endogenous non-price variables (not today)

Assumptions: DGP and Observables

1. (*Equilibrium Selection*) There exists a **unique equilibrium**, or the equilibrium selection rule is such that the same p_t arises whenever the vector $(w_t, x_t, \omega_t, \xi_t)$ is the same.
2. (*Separability of Cost*) The cost function is separable in unobservable shocks:

$$c(q_t, w_t, \omega_t) = \bar{c}(q_t, w_t) + \omega_t.$$

3. (*Known Demand*) The matrix of **demand derivatives is known**, so that D_t is observed.
 4. (*Markup Dependence*) The markup function Δ depends only on endogenous market shares s_t and the matrix of demand derivatives D_t .
- Assumption 4 general (includes most standard static oligopoly models) but not without loss
 - e.g., Bertrand/profit weights: $\Delta = (\mathcal{H}_t \odot D_t)^{-1} s_t$ Cournot: $\Delta = (\mathcal{H}_t \odot (D_t^{-1})') s_t$
 - Demand is known, supply (Δ and \bar{c}) is not

Market Counterfactuals

- Policy changes of interest exogenously change primitive object $a \rightarrow \tilde{a}$
- New market outcomes can be computed by solving fixed point:

$$\tilde{p}_t = \tilde{\Delta}(\tilde{p}_t, \tilde{s}(\tilde{p}_t, \tilde{x}_t, \tilde{\xi}_t)) + \tilde{c}(\tilde{q}_t, \tilde{w}_t) + \tilde{\omega}_t$$

- We can express **counterfactuals** as a map $F(\tilde{p}, \tilde{s}, \cdot)$ from structural objects and exogenous variables to outcomes of interest, e.g., prices, shares, consumer welfare, etc.

Estimating Counterfactuals

- Evaluating the map F requires knowledge of (counterfactual) primitives, exogenous observables, and unobservables
- Researchers use a combination of **data** and **theory/assumptions**
 - We typically estimate functions d and c , and assume Δ with a model of conduct
 - Allows us to specify $\tilde{\xi}, \tilde{\omega}$ and either fix or deterministically change $\tilde{d}, \tilde{c}, \tilde{\Delta}$
- Trade off practicality and data limitations with the dangers of misspecification
 - Estimation of nonparametric models has a curse of dimensionality and requires rich data
 - Misspecification can result in misleading results and subsequent counterfactuals
- Next: a **feasible nonparametric** model of supply

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Nonparametric Model of Supply

- Recall DGP, under assumptions 1-4:

$$p_{jt} = \Delta_j(s_t, D_t, \mathcal{H}_t) + c_j(q_t, w_{jt}) + \omega_{jt}$$

- Without further restrictions, as $q_{jt} = s_{jt}M_t$ for scalar market size M_t :

$$p_{jt} = h_j(s_t, D_t, w_{jt}; \mathcal{H}_t, M_t) + \omega_{jt}$$

for **supply function** $h(\cdot) \equiv \Delta(\cdot) + \bar{c}(\cdot)$

- Notice that formulation of h does **not enforce separability** of cost and markup
 - Extension I: we can assume a model of conduct, and recover costs flexibly (not today)
 - Extension II: we can assume a cost function, and recover markup functions flexibly (not today)
 - Extension III: we can allow separability with more instruments and variation (not today)
- For counterfactuals, find the prices \tilde{p}_t that solve:

$$\tilde{p}_t - \hat{h}(\mathcal{J}(\tilde{x}_t, \tilde{p}_t), D(\tilde{x}_t, \tilde{p}_t), \tilde{w}_t; \tilde{\mathcal{H}}_t) - \hat{\omega}_t = 0$$

- We can define a map F to counterfactuals of interest using estimated objects

Which Counterfactuals?

- This nonparametric supply structure enables a **wide range of counterfactuals**
 - Changes in ownership resulting from mergers, firm/product exit, and product divestment
 - Regulations that alter product characteristics or cost shifters, e.g., fuel economy standards
 - Unit and ad valorem taxes with variation across products and/or markets
- We can measure equilibrium prices, quantities, and changes in consumer welfare (and, in the case of a tax, government revenue and incidence)
- Important **limitations** of our approach:
 - Cannot measure markups and cost levels separately
 - Cannot alter cost or markups separately
 - (Can be addressed w/ extensions of the method)

Identification of h

- Key identification challenge: the s_t and D_t arguments of h are **endogenous**
- We rely on a moment condition with **supply instruments** z_{jt} for identification. Assume:
 5. (*Instrument Exogeneity and Exclusion*) The vector of instruments z_{jt} that satisfies $\mathbb{E}[\omega_{jt} \mid z_{jt}, w_{jt}] = 0$ contains demand shifter(s) $x_{jt}^{(e)}$ that are excluded from the vector w_{jt} .
 6. (*Completeness*) For all functions $B(s_t, D_t, w_t; \mathcal{H}_t)$ with finite expectation, if $\mathbb{E}[B(s_t, D_t, w_t; \mathcal{H}_t) \mid z_{jt}, w_{jt}] = 0$ almost surely, then $B(s_t, D_t, w_t) = 0$ almost surely.
- Result: under 1.-6., h_j is identified for each j
 - Proof follows arguments akin to Berry and Haile (2014)

Instruments and Data Requirements

- **Candidate supply instruments** - need $2J + J(J - 1)/2$, many candidates available
 - Intuitively: rival cost shifters move s_t ; (own and rival) prod. characteristics move D_t
 - Other instruments (e.g., variation in exogenous tax rates) may be available
- Must include demand shifters *excluded* from cost
 - If not, e.g. w/ logit demand, may just recover inverse demand $h = \delta^{-1}$
- Concern: will **variation in the data suffice?**
- We show simulation evidence later on that standard datasets may have enough variation
- Conduct/cost **testing toolkit** allows some flexibility in supply with discipline from theory
 - E.g., procedures in Backus, Conlon, Sinkinson (2021), Duarte et al. (2023)

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- Classic nonparametric estimators (e.g., **series**) are well studied for GMM-type setups
 - For nonparametric IV problem, Ai and Chen (2003); Newey and Powell (2023)
 - See reviews by Carrasco et al. (2007); Chen (2007)
- But, **curse of dimensionality** and **instability** in classical nonparametric estimation
 - Documented in, e.g., Bennett et al. (2019); Bennett and Kallus (2020)
 - (We make no general claim about the usefulness of classical methods)
- We employ the Variational Method of Moments (VMM)
 - VMM accounts for **endogeneity** via moment conditions that standard neural networks ignore
 - We develop a method for our nonparametric supply, adapting VMM + DNN
 - Derive **uniform prediction bands** for prices, shares, consumer surplus, tax revenue
- Method in a nutshell:
 - Deploy two DNNs to learn both optimal instruments and structural supply fcn h

Why Neural Networks and VMM?

- Why **neural network** structures?
 - Learn complex structures and achieve faster convergence rates than nonparametric benchmarks
 - See Bauer and Kohler (2019); Schmidt-Hieber (2020); Kurisu et al. (2025)

- *Example:* Bertrand-Nash with **sparse profit weights**

- Prices (below) can be written in a **sparse tensor decomp** that depends on latent dim $\ell < J$

$$\omega_j(p, c, \mathcal{H}, D) = p_j - c_j - [(\mathcal{H} \odot D')^{-1} s]_j,$$

- Results in Schmidt-Hieber (2020) then implies that DNNs achieve **faster convergence rates**
- Why the **Variational Method of Moments**?
 - In fixed-dimensional parametric settings, VMM **coincides with OWGMM**
 - Inference properties are known; we develop inference for a complex functional of parameters

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How does our method perform?

- I Does it handle **realistic, high-dimensional environments** well with moderate sample sizes?
- II But, what's **inside the black box**?
- III What **range of counterfactuals** can it handle?

Simulations Setup I

- Simple **parametric simulations** to evaluate absolute and relative performance
- For $T = 100; 1,000; 10,000$, market t has either
 - A: $J_t = 2; 3$ with equal probability (small)
 - B: $J = 30$, owned by 5 firms, as US beer market in Miller and Weinberg (2017)
- Training data is random 80% of the dataset (sampled by market)
- Demand is
 - A: simple logit, scalar unobservable ξ_{jt} , three observable x_{jt}
 - B: RCNL as in Miller and Weinberg (2017)

Simulations Setup II: Supply

- We simulate data under two different assumptions on **conduct**...
 - *Bertrand-Nash*: Identity ownership matrix
 - *Profit-Weight*: Off-diagonal weights of $\tau = 0.5$
- ...**cost specifications**...
 - *Linear*: Linear costs with two independent cost shifters w_{jt}
 - *Economies of Scale*: Marginal costs are quadratic in quantities
- ...and policy instruments
 - *Unit Taxes*: Variation in unit taxes across market
 - *Ad Valorem Taxes*: Variation in ad valorem taxes across markets

Comparison of Models

- We recover ω^B, ω^M , and ω^P under Bertrand, Monopoly, and perfect competition
- In the nonparametric supply model, we estimate h and recover ω_{jt}^V :

$$p_{jt} = \hat{h}_j(s_t, D_t, w_{jt}, \mathcal{H}_t) + \omega_{jt}^V$$

- **Supply instruments:** own x_{jt} , rival w_{-jt} , sum of rival x_{jt} , w_{jt}
- We run two types of simulation exercises:
 1. For **A**, **B**: predict in **test sample** (20% of data) w/ estimated function \hat{h} and setting $\omega_{jt}^m = 0$
 2. For **A**: Simulate **counterfactual interventions** in mkt t , predict residual ω_{jt}^m under model m
- For both exercises, used demand and estimated supply to compute fixed point

Test Sample Price Prediction Performance in A

Table 1: MSE Across Models, Bertrand DGP (Small Network), environment A

T	True Model	Standard Models			Flexible	D_t included
		B	M	P		
100	0.90	0.90	578.77	9.41	1.91	No
					1.71	Yes
1,000	0.89	0.89	1022.81	8.17	2.21	No
					1.07	Yes

- Small network has 3×3 hidden layer

Test Sample Price Prediction Performance, Profit-Weight DGP in A

Table 2: MSE Across Models, Profit-Weight DGP (Large Network), environment A

T	True Model	Standard Models			Flexible	D_t included
		B	M	P		
1,000	0.89	3.69	66.42	8.22	1.73	No
					1.74	Yes
10,000	0.96	4.02	77.08	8.79	1.23	No
					1.05	Yes

- Large network has 100×100 hidden layer
- (When we repeat this exercise 100 times w/ different random draws, initialization of NN, we find tight MSE ranges)

Test Sample Price Prediction Performance, Profit-Weight DGP in B

Table 3: MSE Across Models, Profit-Weight DGP, environment B

T	True Model	Standard Models			Flexible ($\#h = 10$)	Flexible ($\#h = 200$)
		B	M	P		
1,000	1.19	3.86	3.76	3.31	2.96	1.58
10,000	1.07	3.96	3.86	3.41	2.74	1.28

- $\#h$ is number of layers

Key Takeaways

- Performance is
 - in A, already reasonable with 100 markets, with 1,000 may be already close to match truth
 - in B, ok with 1,000 markets, probably need closer to 10,000
- Adding derivatives helps, especially in larger samples
- Larger network structure useful to capture complex models of supply, but need more data

Peeking Inside the Black Box: Pass-through

- Key question: How do we **interpret** the flexible \hat{h} we recover?
- A useful object for comparison is the **pass-through matrix** implied by \hat{h}
- To compute pass-through:
 - Pick median post-merger market by inside share from simulations
 - Increase costs c by 10%, loading increases on the residual ω^V
 - Solve for equilibrium prices under different models of conduct
 - Compare price before and after cost change, report price change/cost change

Pass-through Comparison

Table 4: Simulated Pass-through Matrices

Panel A: Bertrand DGP

True Model		Flexible Supply	
0.77	0.17	0.69	0.12
0.10	0.60	0.12	0.63

Panel B: Profit-Weight DGP

True Model ($\kappa = 0.5$)		Flexible Supply	
0.35	-0.31	0.44	-0.21
0.02	0.98	0.01	0.91

- Flexible model learns economics of supply side, **implies pass-throughs close to the true ones**
- (Holds beyond this one market)

Market Counterfactuals

- Results thus far show test sample performance
 - Predict for markets out of the training sample, but from same DGP
- Key aspect of counterfactual prediction: (somewhat) out-of-sample
- Caveat when using our method:
 - As with any nonparametric approach, will struggle too far from the support of the data
- Next set of simulations shows “how far is too far”
- (Throughout, $T = 1,000$, env A, flex model estimated with small network structure)

What Market Counterfactuals?

- Predict prices after **product regulations alter cost shifters** simulations
 - E.g., environmental regulations could increase production costs
- Predict market shares after product **regulation on product characteristics** simulations
 - E.g., bans on menthol in cigarettes or caps on sugar content could change consumption
- Predict welfare changes due to product **entry or exit** simulations
 - E.g., introduction of a new vehicle or merging firms drop products
- Predict welfare changes due to **mergers** simulations
 - E.g., mergers in which existing products have new ownership
- Predict revenues after changes in unit and ad valorem **taxes** simulations
 - Governments could impose taxes on goods

Computational Cost

- Implementation and computation is manageable
- We use the Python package `torch` for all models
- Model fit takes minutes with $T = 100$, an hour for $T = 1,000$, and \sim a day for $T = 10,000$

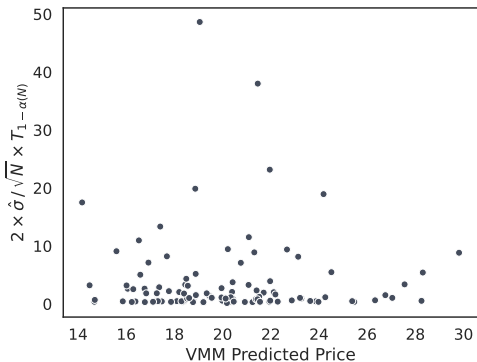
Uncertainty in Counterfactuals

- What about uncertainty in prediction?
- With our VMM estimator, we can compute **standard errors** for counterfactual outcomes
- Two aspects:
 - Computing standard errors is computationally tractable
 - Uncertainty in prediction seems reasonable in simulations
- Exercise: show market-by-market prediction errors for product exit counterfactuals

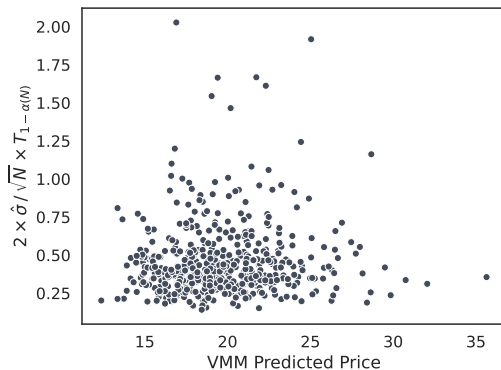
Inference on Counterfactual Prices

Figure 1: Inference on Counterfactual Product Exit Prices

Panel A. Bertrand DGP, $T = 100$



Panel B. Bertrand DGP, $T = 1,000$



- With $T = 1,000$, tight prediction intervals

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Application to Airline Mergers

- Good environment to test our method: airline markets in the US have [rich data](#) from DB1B
 - Fares, passenger counts, distances, carrier identifiers, etc.
 - Origin and destinations of trips
 - Several large mergers in sample
 - 1mln+ obs pooling quarterly data 2005-2019, we use $\sim 10,000$ pre-merger markets for VMM
 - Previous merger retrospectives (Peters, 2006)
- Estimate simple nested logit demand model
- Goal: predict unilateral price effects of American-US Airways merger in Q4 2013
 - Zoom in on markets that move from 3 \rightarrow 2 firms post-merger
 - Treated markets are markets in which both merging firms are present
- (We abstract from many interesting aspects of the industry here...)

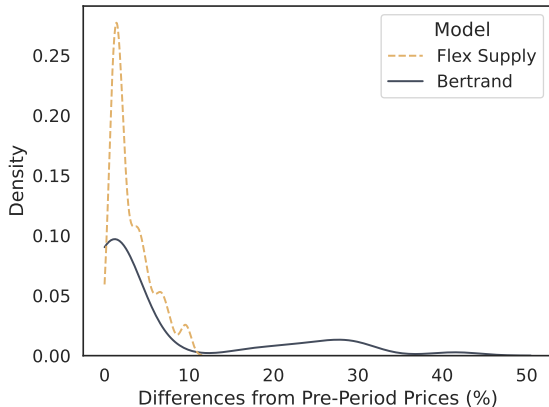
Mergers

Prices

Demand

Fit

Figure 2: Predicted Price Change Distribution

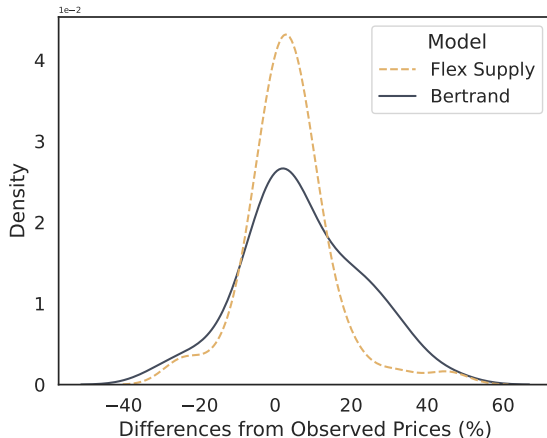


Model	Price Increases (%)	
	Median	Mean
Bertrand	1.45	6.66
VMM	2.05	2.16

- In theory, our flexible supply model could predict price decreases but it doesn't here

Merger Simulation: Comparing Predicted and Observed Post-merger Prices

Figure 3: Merger Simulation Comparison



Model	MSE
Bertrand	365.71
VMM	66.93

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- Market counterfactuals crucially depend on the supply model
 - We develop a nonparametric model of supply for a range of counterfactuals
 - Estimation technique uses deep learning + objective function with instruments
 - Inference procedure quantifies uncertainty of a complex functional
- Simulation exercises and an empirical application showcase the method
 - Outperform misspecified models across a host of counterfactuals
 - Merger simulation in the airline industry outperforms the standard merger simulation toolkit

Thank You!

- Our model and assumptions imply a moment condition for the structural supply function:

$$\mathbb{E}[p_{jt} - h_j(s_t, D_t, w_t, \mathcal{H}_t) \mid z_t, w_t] = 0$$

- The **VMM estimator** (Bennett and Kallus, 2023) for our setting is:

$$\hat{\theta}_N = \operatorname{argmin}_{\theta \in \Theta} \sup_{f \in \mathcal{F}_N} \frac{1}{TJ} \sum_{j,t} f(z_{jt})^T \omega_{jt}(\theta) - \frac{1}{4TJ} \sum_{j,t} (f(z_{jt})^T \omega_{jt}(\tilde{\theta}_N))^2 - R_N(f, h)$$

$$\text{s.t. } \omega_{jt}(\theta) = p_{jt} - h_j(s_t, D_t, w_t, \mathcal{H}_t; \theta,) \quad \forall j \in J$$

- $\tilde{\theta}_N$ is preliminary estimate
- Both f and h are **neural networks**, allowing flexible controls of model complexity
- Quadratic term motivated by **optimal weighting** of making each moment condition zero
- $R_N(\cdot)$ is **regularizer** that penalizes complexity

- We establish **simultaneous confidence intervals** for d predicted or counterfactual prices \hat{h} :

$$\sqrt{N}(\hat{h} - h_0) \xrightarrow{d} N(0, \nabla_{\theta'} h_0 \Omega_0^{-1} \nabla_{\theta'} h_0^T)$$

- Prices are not the only counterfactual of interest in economics research
 - E.g., quantities, consumer surplus, government revenue
- Assuming smoothness of the counterfactual map F in prices, we establish:

$$\sqrt{N}(F(\hat{h}) - F(h_0)) \xrightarrow{d} N(0, \nabla_h F(h_0) \nabla_{\theta} h_0 \Omega_0^{-1} \nabla_{\theta} h_0^T \nabla_h F(h_0)^T)$$

- We can **quantify uncertainty** on other economic objects of interest
 - Inference is possible for product-level, market-level, and aggregate objects

Inference: Simplest Case ($d = 1$) [Back](#)

- Note that $\nabla_{\theta'} h(\theta_0)$ is $d \times b$; in the simplest case, suppose that $d = 1$
- Lemma 9 in Bennett and Kallus (2023) states that for any $\beta \in \mathbb{R}^b$, we have:

$$\beta^T \Omega_0^{-1} \beta = -\frac{1}{4} \inf_{\gamma \in \mathbb{R}^b} \sup_{f \in \mathcal{F}} \left\{ \mathbb{E}[f(Z)^T \nabla_{\theta} \omega(X; \theta_0) \gamma] - \frac{1}{4} \mathbb{E}[(f(Z)^T \omega(X; \theta_0))^2] - 4\gamma^T \beta - R_N(f, h) \right\} \quad (1)$$

- Take $\beta = \nabla_{\theta} h_x(\theta_0)$ and the above solution to the optimization problem becomes:

$$\sigma_x^2 = \nabla_{\theta} h_x(\theta_0) \Omega_0^{-1} \nabla_{\theta} h_x(\theta_0)^T$$

- This is the asymptotic variance for $\sqrt{N}(h_x(\hat{\theta}_N) - h_x(\theta_0))$
 - $\nabla_{\theta} h_x(\theta_0)$ can be difficult to compute analytically
 - Numerical differentiation can be employed (e.g., Hong et al. (2015))
 - Expectations can be replaced by sample means, $\hat{\theta}_N$ can be used in place of θ_0
 - These together yield a feasible version of Equation (1) which provides an estimator $\hat{\sigma}_x^2$ for σ_x^2

- The approach above cannot obtain a covariance matrix when $d \geq 2$
- Holm's Step-Down procedure using the estimates for $\hat{\sigma}_{x_j}^2$ and $h(\hat{\theta})$ for each $j = 1, \dots, d$
- The set of critical values T_α is known for significance levels $\frac{\alpha}{d+1-k}$ and $k = 1, \dots, d$
 - We can use a folded normal distribution with $t = 1$ to account for bias
- For any ordering of x and fixed ordering T_α , we can compute the confidence interval:

$$h_x(\hat{\theta}) \pm N^{-\frac{1}{2}} \hat{\sigma}_x T_\alpha$$

- We compute this for all permutations of $j = 1, \dots, d$, resulting in $d!$ permutations of x
- This is because we must consider any possible ordering of the p-values of x_1, \dots, x_d

1. Estimate $\hat{\sigma}_{x_j}^2$ for $\sigma_{x_j}^2$ for $j \in \{1, \dots, d\} \equiv J$ by solving the feasible version of Equation (1)
2. Fix values $T_\alpha = \{T_{\alpha_k} : k = 1, \dots, d\}$ where $\alpha_k = \frac{\alpha}{d+1-k}$
3. For each permutation \tilde{J} of J :
 - 3.1 Arrange values \tilde{x} and $\hat{\sigma}_{\tilde{x}}$ with permuted indices \tilde{J}
 - 3.2 Construct bounds as $h_{\tilde{x}}(\hat{\theta}) \pm n^{-\frac{1}{2}} \hat{\sigma}_{\tilde{x}} T_\alpha$ with fixed T_α
4. Simultaneous confidence interval as the union of $2 \times d \times d!$ linear constraints from Step (3)

Regulation of Cost Shifters

- For all counterfactuals, use $\text{RMPSE} \simeq \text{avg magnitude of percentage errors}$
- Implementation: add 1 to $w_{jt}^{(1)} \sim U(0, 1)$

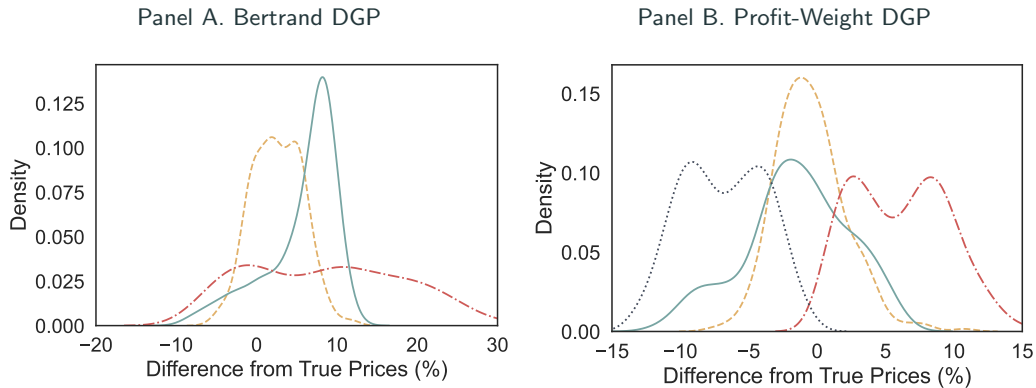
Table 5: RMPSE in Prices for Cost Shifter Regulation

Fitted Model	Panel DGPs			
	A. Bertrand	B. Profit-Weight	C. Bertrand (Scale)	D. Profit-Weight (Scale)
--- Bertrand (Scale)	--	--	--	5.0
..... Bertrand (Const.)	--	5.1	3.5	6.0
--- Monopoly	10.2	5.1	11.6	6.5
— Perf Comp	5.0	6.3	5.0	3.8
--- Flex Supply	2.8	2.5	3.5	3.0

- Good absolute and relative performance for a fairly out-of-sample counterfactual

Regulation of Cost Shifters

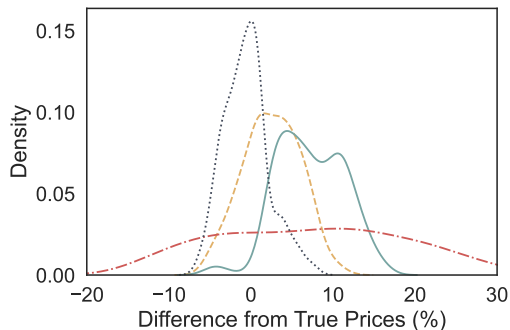
Figure 4: Regulation of Cost Shifters



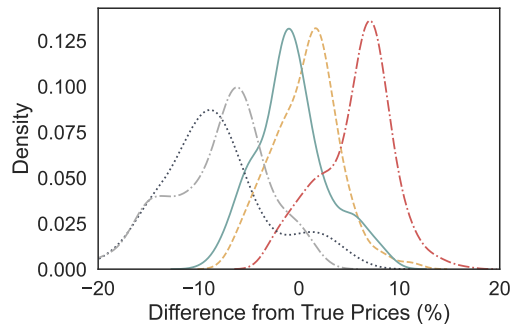
Regulation of Cost Shifters with Economies of Scale

Figure 5: Regulation of Cost Shifters

Panel C. Bertrand DGP, Economies of Scale



Panel D. Profit-Weight DGP, Economies of Scale



Regulation of Product Characteristics

- Implementation: add 1 to $x_{jt}^{(1)} \sim U(0,1)$

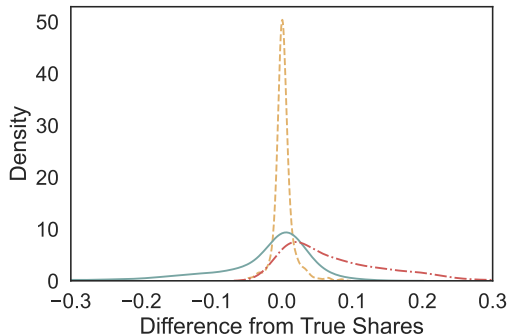
Table 6: RMPSE in Shares for Regulation of Product Characteristics

Fitted Model	Panel DGPs			
	A. Bertrand	B. Profit-Weight	C. Bertrand (Scale)	D. Profit-Weight (Scale)
--- Bertrand (Scale)	--	--	--	20.3
..... Bertrand (Const.)	--	21.0	10.2	28.3
--- Monopoly	31.9	22.3	15.9	11.1
--- Perf Comp	32.9	57.7	12.7	16.4
--- Flex Supply	5.0	6.6	6.6	9.2

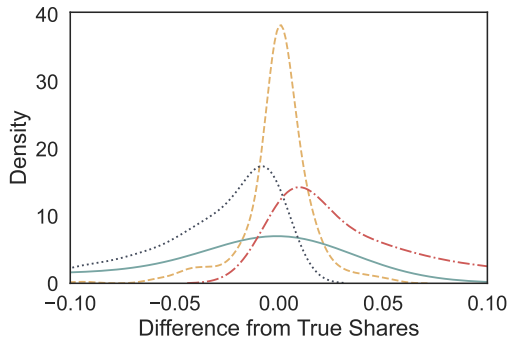
Regulation of Product Characteristics

Figure 6: Regulation of Product Characteristics

Panel A. Bertrand DGP



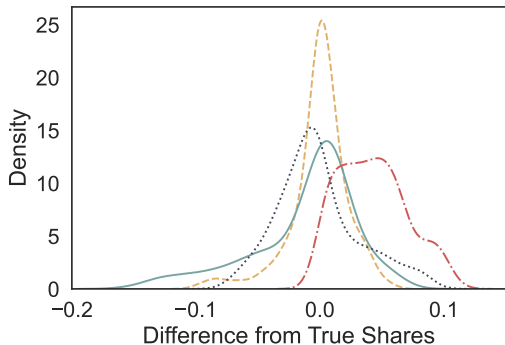
Panel B. Profit-Weight DGP



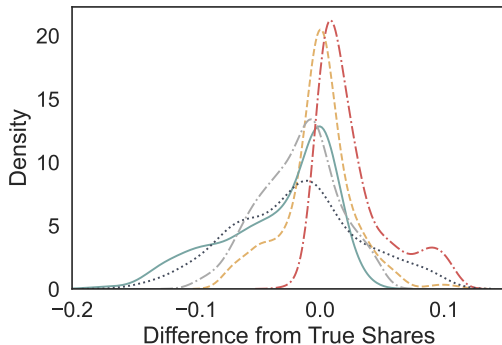
Regulation of Product Characteristics with Economies of Scale

Figure 6: Regulation of Product Characteristics

Panel C. Bertrand DGP, Economies of Scale



Panel D. Profit-Weight DGP, Economies of Scale



- Predicting shares seems somewhat harder

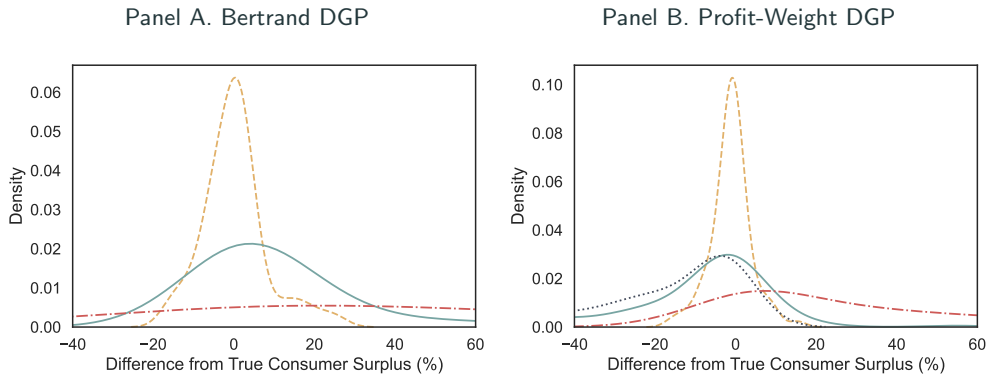
Product Exit

- Implementation: drop a product

Table 7: RMPSE in Consumer Surplus for Product Exit

Fitted Model	Panel DGPs			
	A. Bertrand	B. Profit-Weight	C. Bertrand (Scale)	D. Profit-Weight (Scale)
— · — Bertrand (Scale)	--	--	--	25.0
····· Bertrand (Const.)	--	23.9	7.7	23.2
— · — Monopoly	135.4	47.8	239.4	79.6
— — — Perf Comp	40.1	22.1	19.1	19.6
— — — Flex Supply	7.8	5.0	5.4	5.3

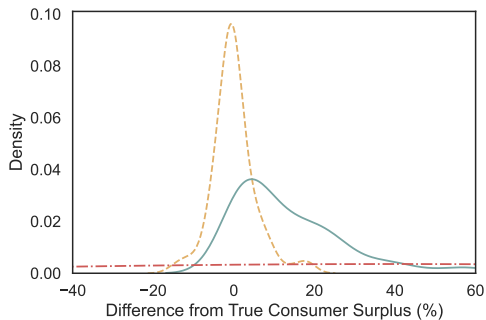
Figure 7: Product Exit



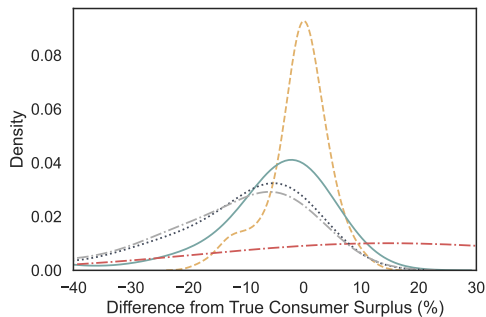
Product Exit with Economies of Scale

Figure 7: Product Exit

Panel C. Bertrand DGP, Economies of Scale



Panel D. Profit-Weight DGP, Econ. of Scale



Multi-product Merger Simulation

- Implementation: change ownership of one product

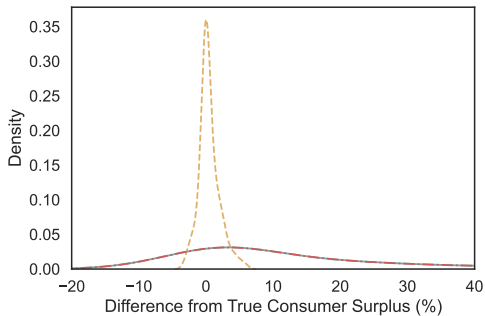
Table 8: RMPSE in Consumer Surplus for Mergers

Fitted Model	Panel DGPs			
	A. Bertrand	B. Profit-Weight	C. Bertrand (Scale)	D. Profit-Weight (Scale)
— · — Bertrand (Scale)	--	--	--	4.8
····· Bertrand (Const.)	--	5.0	0.8	4.9
— · — Monopoly	23.1	10.4	20.8	9.3
— — — Perf Comp	23.1	10.4	20.8	9.3
— — — Flex Supply	1.5	2.4	4.9	3.9

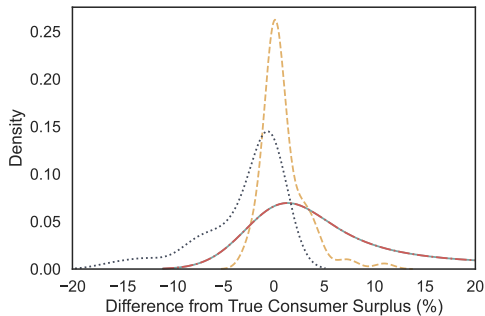
Multi-product Merger Simulation

Figure 8: Merger Simulation

Panel A. Bertrand DGP



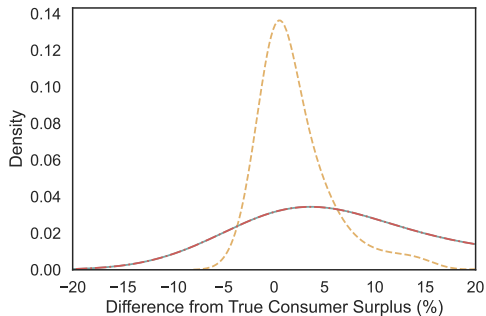
Panel B. Profit-Weight DGP



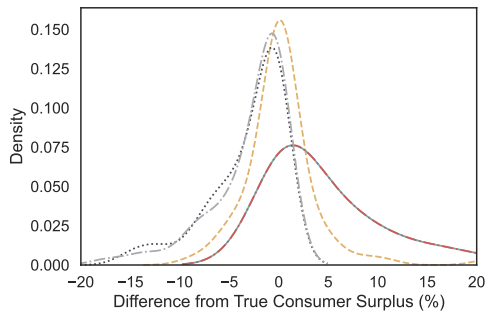
Multi-product Merger Simulation with Economies of Scale

Figure 9: Merger Simulation

Panel C. Bertrand DGP, Economies of Scale



Panel D. Profit-Weight DGP, Econ. of Scale



Laffer Curves

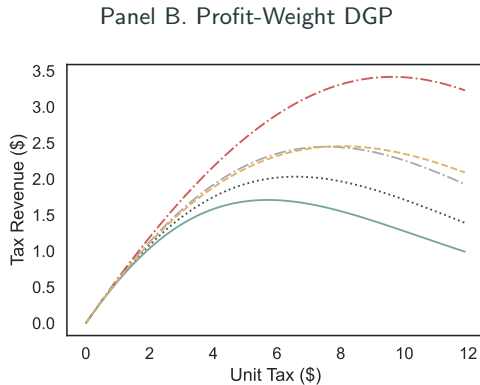
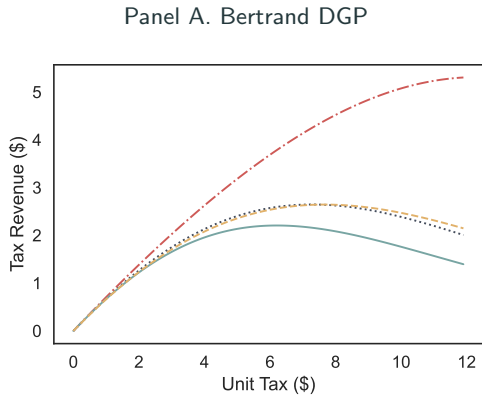
- Implementation: augment DGP with unit and ad valorem taxes, generate some variation in tax rate in training data

Table 9: MSE in Government Revenue for Laffer Curves

Fitted Model	Panel DGPs			
	A. Bertrand (Unit)	B. Profit-Weight (Unit)	C. Bertrand (AV)	D. Profit-Weight (AV)
..... Bertrand	--	0.13	--	0.27
--- Monopoly	2.88	0.54	16.67	2.04
— Perf Comp	0.17	0.45	0.96	1.34
--- Flex Supply	0.00	0.00	0.03	0.05

Laffer Curves for Unit Taxes

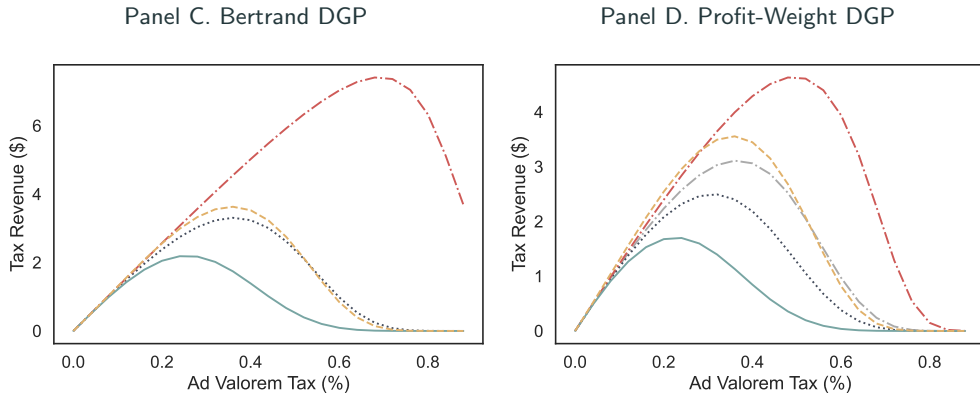
Figure 10: Laffer Curves



- In training data unit tax is $U[4, 8]$

Laffer Curves for Ad Valorem Taxes

Figure 11: Laffer Curves



- In training data ad valorem tax is $U[0, 0.8]$

Figure 12: HHI in the Airline Industry

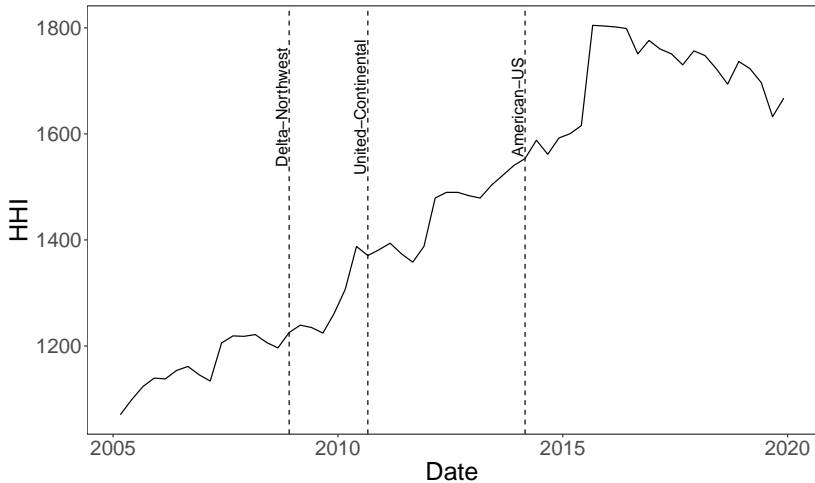
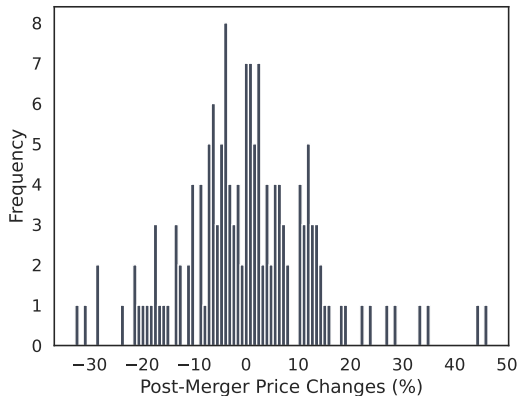


Figure 13: Price Change Distribution



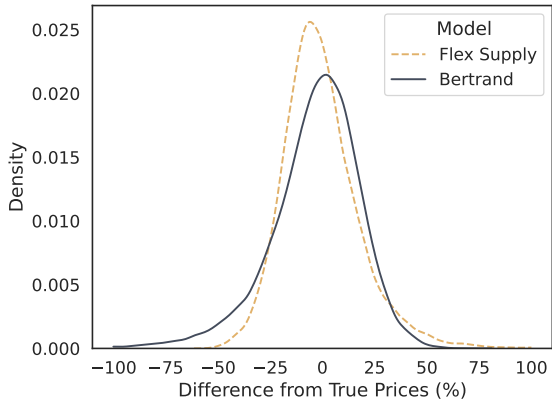
- Price changes after the AA-US merger in 3 \rightarrow 2 markets

Table 10: Demand Estimates

	$\log(s_{jt}) - \log(s_{0t})$
Average Fare	-0.0048*** (0.0004)
$\log(S_t)$	0.8356*** (0.0133)
Share Nonstop	0.4030*** (0.0282)
Average Distance (1,000's)	-0.4881*** (0.0498)
Average Distance ² (1,000's)	0.0485*** (0.0045)
$\log(1 + \text{Num. Fringe})$	-0.2642*** (0.0057)
R^2	0.94238
Observations	1,283,472
Own-price elasticity	-5.1652
Origin-destination fixed effects	✓

- Elasticities broadly in line with literature (e.g., Berry and Jia, 2010)

Figure 14: Model Comparison



Model	Sample	MSE
Bertrand	All	1949.59
VMM	All	1242.95
Bertrand	Train	1932.70
VMM	Train	1235.39
Bertrand	Test	2016.33
VMM	Test	1272.82

- Reduction of $\sim 40\%$ in passenger-weighted MSE relative to Bertrand

Figure 15: Width of Confidence Intervals

