Platform Switching Policies under Collusion: Unintended Consequences for Digital Platforms

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Abstract

This paper examines the theoretical implications of policies facilitating cross-platform participation in digital markets. We develop a model of duopoly platform competition analyzing both price-setting (e.g., ride-hailing) and fee-setting (e.g., vacation rentals) mechanisms. We find that when the platforms are in static competition, reducing switching costs generally benefits consumers; but when platforms collude, the effects can be reversed dramatically. Notably, policies facilitating seller cross-platform participation may reduce buyer and seller welfare under collusion.

1 Introduction

The growing economic importance of digital platforms has sparked intense interest among policymakers and competition authorities in regulating these markets. A particular focus has been on reducing barriers that prevent users from accessing and switching between different platforms. This strategy is widely viewed as pro-competitive, with the potential to reduce market concentration and improve outcomes for consumers. For instance, the European Commission's Digital Markets Act includes provisions to reduce technical, economic and contractual barriers to platform switching and cross-platform participation.[1] Similarly, the UK Competition and Markets Authority (CMA) considers consumers' ability to freely access multiple platforms as an effective check against incumbent platforms becoming entrenched in their market positions.[2]

In this paper, we explore the theoretical implications of policies facilitating cross-platform participation on platforms that intermediate transactions between buyers and service providers. Examples include ride-sharing, vacation rentals, and on-demand services. These platforms, following years of

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[1]See European Commission. (2022). The Digital Markets Act: Ensuring Fair and Open Digital Markets. The DMA contains multiple provisions aimed at reducing barriers to platform switching and cross-platform access. These include prohibitions on technical restrictions that limit switching, requirements for effective data portability, and mandates for interoperability of certain platform features.

[2]CMA, Online Platforms and Digital Advertising Market Study, Final Report, 2020.

rapid growth, now represent a significant portion of economic activity. Our analysis highlights three factors that characterize these marketplaces and potentially shape the effects of platform switching policies.

First, price determination varies across different types of platforms. *On-demand platforms* directly set prices for buyers while adjusting seller prices (and platform fees) to meet demand. For instance, BetterHelp establishes a fixed weekly subscription fee for clients, while Uber sets a base fare for riders, which may be adjusted through surge pricing but remains a key reference point. In contrast, *peer-to-peer platforms* with heterogeneous products, such as Airbnb, operate differently. These platforms don't set prices directly. They instead determine the commission rate or fee, which is the fraction of each sale they retain. Market competition then establishes the final prices for buyers and sellers. This distinction in pricing mechanisms significantly influences how these platforms operate and compete.

Second, the concept of cross-platform participation (and how it relates to the standard notion of multi-homing) in these markets is nuanced. While users of both on-demand and peer-to-peer platforms can easily switch between platforms over time, they commit to a single platform for each specific transaction. For instance, a BetterHealth counselor is exclusively committed to that platform for the duration of an appointment. Similarly, an Instacart shopper selects one app for each shopping trip, even if they have multiple delivery apps installed. In the realm of vacation rentals or car-sharing, property owners or car owners can't double-book a unit on multiple platforms for the same time slot. Thus, while agents maintain access to multiple platforms in aggregate, they single-home for a given transaction.

Third, these markets tend to have a high degree of market concentration. For example, the combined market share of Uber and Lyft in the U.S. ride-sharing market is virtually 100%,[3] while Airbnb and Vrbo dominate the homeshare market with about 95% combined share.[4]

To capture economic interactions in these environments, we develop a rich model of duopoly platform competition incorporating the specific features of these markets. We consider two pricing mechanisms: one where platforms choose prices for buyers (with seller prices and platform fees determined by market clearing), and another where firms choose proportional fees for sellers (with both buyer and seller prices determined through market clearing). In either scenario, the platforms move first and set fees or prices. Then, buyers and sellers with differentiated tastes for the two platforms decide first where to transact, and then choose quantities. The choice of platform depends on the expected surplus from the transaction, but also on a travel cost that each buyer and seller has to incur.

Since high concentration makes collusion a substantial threat, we consider the possibility that platforms' actions could either correspond to a competitive or to a collusive equilibrium. We characterize these equilibria for on-demand and peer-to-peer platforms separately. While the joint

[3] See https://www.statista.com/statistics/910704/market-share-of-rideshare-companies-united-states/, accessed October, 2024.

[4] See https://seekingalpha.com/news/3846023-airbnb-increases-market-share-in-latest-read-from-m-science, accessed October, 2024.

profit-maximizing outcomes are the same for these two types of platforms, both the competitive and any constrained collusive equilibria may differ.

Our model, while not analytically solvable, yields insights through numerical simulations. We examine a "platform switching counterfactual" by reducing platform travel costs for buyers or sellers, simulating a regulatory intervention to reduce barriers to platform switching. While travel costs in our model technically represent horizontal differentiation, we interpret reductions in these costs as capturing the effect of policies that make it easier for users to choose between platforms on a transaction-by-transaction basis.[5] Focusing on buyer surplus, we find that in a static Nash equilibrium, reducing buyer travel costs increases buyer surplus under both pricing models. Reducing seller travel costs also increases buyer surplus in the price-setting model, but has a small, nonmonotonic effect in the fee-setting model.

We then explore the possibility of platform collusion using a grim trigger strategy in an infinitely-repeated game. The sustainability of collusion depends on the trade-off between short-term gains from undercutting and long-term losses from reverting to the Nash equilibrium; since the economic environment influences both, its impact on collusive possibilities is complex. Under collusion, we find that reducing buyer travel costs still increases buyer surplus in the price-setting model, but has minimal, nonmonotonic effects in the fee-setting model. Notably, reducing seller travel costs in the price-setting model now dramatically decreases both buyer and seller surplus — a reversal from the competitive scenario. Similar reversals occur for other welfare measures.

The high concentration observed in many digital platform markets (with duopolies holding 95-100% market share in ride-sharing and homeshare markets) makes collusion a plausible concern for regulators. Recent research has also highlighted the potential for algorithmic pricing to facilitate tacit collusion in digital markets (Calvano, Calzolari, Denicolo, and Pastorello, 2020), further motivating our analysis of collusive outcomes. Our findings suggest that if platforms are colluding, policy interventions could have unintended, even opposite, effects compared to competitive markets: facilitating platform switching, especially for sellers, can sometimes reduce buyer welfare. Our analysis underscores the theoretical ambiguity of platform switching policies and their dependence on specific market conditions. Moreover, the model highlights that understanding platform conduct is crucial to predicting the impact of platform switching policies.

The literature on two-sided markets and network effects has grown substantially in recent decades (see, e.g., the excellent survey by Jullien, Pavan, and Rysman, 2021). Seminal theoretical contributions include Rochet and Tirole (2003) and Armstrong (2006), which provide different frameworks for modeling two-sided markets. Recent studies have built upon these foundations to examine multi-homing in specific contexts. Within the Rochet and Tirole (2003) framework, Teh, Liu, Wright, and Zhou (2023) study competition between multiple ride-sharing platforms when both sides of the market are able to multi-home. Within the Armstrong (2006) framework, Bryan and Gans (2019) explore how platforms might compete through pricing and wait-time management.

[5] This differs from classic switching cost models that focus on consumer inertia (Klemperer, 1995), as our framework emphasizes the impact of reducing barriers to platform selection for each individual transaction.

Our model synthesizes elements from these approaches. We adopt from Rochet and Tirole (2003) the concept of controlling network effects through per-transaction prices or fees, while incorporating from Armstrong (2006) the notion of agent preferences through transportation costs. However, our focus on platform switching as opposed to standard multi-homing diverges from much of the existing literature. While previous studies often define multi-homing as simultaneous participation on multiple platforms,[6] we consider a more nuanced scenario. In our model, agents can switch between platforms across transactions but commit to a single platform for each specific transaction. This approach aligns with the realities of many on-demand and peer-to-peer markets, where simultaneous use of multiple platforms for a single transaction is often impractical or impossible. This distinction is crucial for understanding the dynamics of platform competition in these rapidly evolving markets.

Our work is related to papers that have proposed frameworks for understanding the incentives to collude within two-sided markets. Jullien and Sand-Zantman (2021) provide a comprehensive overview of this literature; more recently, Peitz and Samkharadze (2022) study collusion among non-differentiated platforms. Notable studies in this area include Ruhmer (2010) and Lefouili and Pinho (2020), who examine collusion's effects within Armstrong's model, and Dewenter, Haucap, and Wenzel (2011), who investigate collusion in newspaper markets with multi-homing readers and advertisers. These studies consistently find that full collusion maximizes platform profits, but the welfare implications are ambiguous and depend on network externalities. Collusion can sometimes improve welfare for certain market participants, particularly when strong network effects are present. Our approach differs by focusing on how horizontal differentiation or platform switching (as measured by travel costs) affects market outcomes under both competition and collusion. We demonstrate that in some cases, collusion can reverse the impact of reduced travel costs, potentially undermining the intended effects of regulatory interventions designed to enhance competition by encouraging platform switching and cross-platform participation.

Finally, the paper also relates to the broader literature showing the ambiguous effects of product differentiation on collusion (e.g., Deneckere, 1983; Chang, 1991; Rothschild, 1992; Häckner, 1994; Powers, 1998; Thomadsen and Rhee, 2007). Previous work has established that product differentiation creates a trade-off for collusion, which is also present in our results: less differentiation intensifies competition under Nash reversion but also increases the gains from collusion. However, our platform setting shows how this trade-off manifests uniquely in two-sided markets, where the relationship between differentiation and collusion hinges on the platform's pricing mechanism. This distinction is absent in traditional differentiated product markets and highlights the importance of understanding specific platform business models when evaluating policy interventions.

[6] An exception is the setup considered in Athey, Calvano, and Gans (2018), where consumers "multi-home" by allocating a fixed budget of attention across two publishers.

2 Model

2.1 Overview – Players and Timing

We now introduce the basic structure common to both the price-setting and the fee-setting game. There is a measure of buyers normalized to 1, a measure M of sellers, and two platforms where buyers and sellers can trade with each other. Both buyers and sellers are distributed uniformly along a Hotelling line, with one platform at each end. The timing of the game is as follows:

1. First, the two platforms each simultaneously set either their buyer price or platform fee, explained further below.

2. Second, each buyer and each seller chooses a platform, based on the two platforms' policies and correct expectations about what the other buyers and sellers will do.

3. Third, trade occurs at each platform between the buyers and sellers who have chosen that platform.

2.2 Buyer and Seller Payoffs

Each buyer or seller can only trade on one platform. Once they arrive at a platform, buyers have a linear demand function

$$Q = 1 - \theta P^D$$

where P^D is the price they face at that platform, and therefore earn buyer surplus of $u = \frac{1}{2\theta}(1 - \theta P^D)^2$. Sellers have a linear supply function

$$Q = \gamma P^S$$

where P^S is the price they receive, and therefore earn producer surplus of $\frac{1}{2}\gamma(P^S)^2$. In equilibrium, $P^D > P^S$, since the platforms will choose policies that earn positive profits.

In addition to this surplus, both buyers and sellers have horizontally differentiated tastes for the two platforms, modeled as "travel costs". The buyer at location $x \in [0, 1]$ faces "travel cost" tx from choosing the first platform, and $t(1-x)$ from choosing the second platform, and additionally earns a constant utility U from either platform which is large enough to ensure that in equilibrium, all buyers are served. Thus, the buyer at location x earns $u_1 - tx + U$ from platform 1, and $u_2 - t(1 - x) + U$ from platform 2, where $u_i = \frac{1}{2\theta}(1 - \theta P_i^D)^2$ is the gross buyer surplus they anticipate at platform i.

Similarly, the seller at location $x \in [0, 1]$ faces travel cost Tx from choosing the first platform, and $T(1 - x)$ from choosing the second, and additionally earns a constant utility U' from either which is sufficiently large so that in equilibrium, all sellers sell. The seller at location x therefore earns $\frac{1}{2}\gamma(P_1^S)^2 - Tx + U'$ from platform 1, or $\frac{1}{2}\gamma(P_2^S)^2 - T(1 - x) + U'$ from platform 2, if they expect to face prices P_1^S and P_2^S.

2.3 Platform Payoffs and Actions

The platforms both seek to maximize their revenue, which is the volume of trade occurring at platform i times the difference between P_i^D (the price paid by buyers) and P_i^S (the price paid to sellers). Platforms compete with each other in one of two ways.

In one version of the model, platforms directly set the price faced by buyers, P_i^D, and implicitly guarantee they will pay sellers enough to fill the buyer demand at their platform. This seems like a plausible model for settings where the product is fairly homogeneous and the price facing buyers is fairly focal, such as the competition between Uber and Lyft. While prices are sometimes determined by "market clearing" due to surge pricing, if Uber or Lyft changed the "baseline price" paid for rides, this would be a clear and noticeable change.

In the second version of the model, platforms instead set their fee level - the fraction of buyer payments that the platform keeps, rather than passing on to the sellers. That is, each platform sets a fee level f_i. Then, given the buyers and sellers who come to the platform, prices are determined by market-clearing and the constraint that $P_i^S = (1 - f_i)P_i^D$. This seems like a better assumption when products are heterogeneous, and the platform therefore can't control prices effectively, such as the competition between AirBnB and Vrbo: Vrbo could try to undercut AirBnB by announcing lower fees.

As noted above, once the two platforms set their "policies" (either buyer prices or fee levels), buyers and sellers form beliefs about market conditions at each platform and choose the one they anticipate higher surplus from; given the traders who arrive at each platform, prices are determined, trade occurs, and payoffs are realized.

3 Analyzing The Price-Setting Model

First, we consider the on-demand platforms model, in which the platforms directly set the buyer-facing prices P_i^D, then let the prices paid to sellers P_i^S adjust to clear the market. As usual, we solve the game by backward induction.

3.1 Stage 3: Price Determination and Trade

Suppose buyer prices P_1^D and P_2^D were set in the first stage, and measures $N_i \leq 1$ of buyers and $M_i \leq M$ of sellers have chosen platform i. The buyer price P_i^D is already fixed; the seller price P_i^S is determined by market clearing, which requires the supply at platform i to match the demand, or

$$M_i(\gamma P_i^S) \quad = \quad N_i(1 - \theta P_i^D) \quad \rightarrow \quad P_i^S \quad = \quad \frac{N_i}{\gamma M_i}(1 - \theta P_i^D).$$

Buyer surplus at platform i (gross of travel costs) is $u_i = \frac{1}{2\theta}(1-\theta P^D)^2$, producer surplus is $\pi_i = \frac{1}{2}\gamma(P_i^S)^2 = \frac{\gamma}{2}(\frac{N_i}{\gamma M_i}P_i^D)^2$, and the platform's revenue is

$$\Pi_i \quad = \quad Q_i(P_i^D - P_i^S) \quad = \quad N_i(1-\theta P_i^D)\left(P_i^D - \frac{N_i}{\gamma M_i}(1-\theta P_i^D)\right).$$

3.2 Stage 2: Buyers and Sellers Choose a Platform

Next, we consider the second stage: buyers' and sellers' choice of a platform. Since buyer prices P_i^D are set directly in stage one, buyers know exactly what surplus they can achieve at each platform, and choose accordingly: the buyer at location $x \in [0,1]$ prefers platform 1 to platform 2 if

$$\begin{aligned} \frac{1}{2\theta}(1-\theta P_2^D)^2 - t(1-x) + U \quad &\leq \quad \frac{1}{2\theta}(1-\theta P_1^D)^2 - tx + U \\ x \quad &\leq \quad \frac{1}{2} + \frac{1}{4t\theta}\left[(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2\right]. \end{aligned}$$

Thus, assuming both platforms price to attract a positive measure of buyers (as they will in equilibrium), prices P_1^D and P_2^D will lead to a measure

$$\alpha \quad = \quad \frac{1}{2} + \frac{1}{4t\theta}\left[(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2\right]$$

of buyers choosing platform 1, and $1-\alpha$ buyers choosing platform 2.[7]

Now, let β be the fraction of sellers choosing platform 1, so that a measure βM of sellers sell to the α buyers there. Market clearing requires $P_1^S = \frac{N_1}{\gamma M_1}(1-\theta P_1^D) = \frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)$, and sellers who choose platform 1 will therefore expect producer surplus of $\frac{\gamma}{2}\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)^2$.

Similarly, sellers choosing platform 2 will expect producer surplus of $\frac{\gamma}{2}\left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)^2$. As a result, sellers will choose platform 1 when

$$\frac{\gamma}{2}\left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)^2 - T(1-x) + U' \leq \frac{\gamma}{2}\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)^2 - Tx + U'.$$

Simplifying, this is when

$$x \leq \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)^2 - \left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)^2\right].$$

Assuming an interior solution, the fraction β of sellers choosing platform 1 will therefore be the

[7] If $(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2 \geq 2t\theta$, all buyers prefer platform 1 and $\alpha = 1$; if $(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2 \leq -2t\theta$, all buyers prefer platform 2 and $\alpha = 0$.

unique solution to

$$\beta = \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)^2 - \left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)^2\right]$$

with α as defined above. Assuming $1-\theta P_1^D$ and $1-\theta P_2^D$ are both strictly positive and $\alpha \in (0,1)$, the right-hand side of this last expression goes to $+\infty$ as $\beta \to 0$ and $-\infty$ as $\beta \to 1$, so a solution exists; the right-hand side is decreasing in β, so the solution is unique.

Thus, for P_1^D and P_2^D both less than $\frac{1}{\theta}$ and such that $(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2 \in (-2t\theta, 2t\theta)$, a choice of buyer prices leads to a unique determination of how many buyers and how many sellers choose each platform (which leads to a unique determination of seller price), so platform payoffs are well-defined as a function of (P_1^D, P_2^D).[8]

3.3 Stage 1: Platforms Set Buyer Prices

In the first stage, the platforms choose buyer prices P_i^D, knowing the market shares and eventual seller prices they will lead to.

Symmetric Nash Equilibrium

Consider platform 1's problem of choosing P_1^D to maximize

$$\Pi_1 \quad = \quad Q_1(P_1^D - P_1^S)$$

where Q_1 is the volume of trade at platform 1. We can write Q_1 as the measure of buyers who show up, α, times the demand of each one, which is $1-\theta P_1^D$. And from market clearing above, we know $P_1^S = \alpha(1-\theta P_1^D)$. So we can rewrite Π_1 as

$$\Pi_1 \quad = \quad \alpha(1-\theta P_1^D)\left(P_1^D - \frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)$$

where α and β are functions of P_1^D and P_2^D as given above. The payoffs to platform 2 are similarly

$$\Pi_2 \quad = \quad (1-\alpha)(1-\theta P_2^D)\left(P_2^D - \frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right).$$

Since α and β are fairly complex (and we don't have a closed-form expression for β), we can't simply plug them into Π_1 and Π_2 and fully analyze the first-stage game. However, we can find a necessary condition for symmetric Nash equilibrium, as follows: we take the derivative of Π_i with respect to P_i^D, including the derivatives of α and β, and set it equal to zero at $P_1^D = P_2^D = P_n$ (which also implies $\alpha = \beta = \frac{1}{2}$). This leads to the following condition:

[8] A choice of $P_i^D \geq \frac{1}{\theta}$ will never be optimal, as it induces zero trade and therefore zero revenue. Choices of P_i^D such that $\alpha = 0$ or $\alpha = 1$ never occur in equilibrium, but will need to be accounted for as possible deviations by one platform.

Proposition 1. *If $P_1^D = P_2^D = P_n$ is a symmetric Nash equilibrium of the price-setting game, then P_n must satisfy*

$$
\begin{aligned}
t\gamma M\theta P_n = & -\gamma M P_n(1-\theta P_n)^2 + (1-\theta P_n)^3 + t\gamma M(1-\theta P_n) + 2t\theta(1-\theta P_n) \\
& + \frac{T\gamma M^2 - t\theta}{T\gamma M^2 + 2(1-\theta P_n)^2}(1-\theta P_n)^3
\end{aligned} \tag{1}
$$

Note (1) is a fifth-order polynomial in P_n, and quite tractable computationally. Also note that a solution always exists. At $P_n = 0$, the left-hand side is 0 and the right-hand side is easily shown to be positive; at $P_n = \frac{1}{\theta}$, the right-hand side is 0 and the left-hand side is positive; both sides are continuous in P_n, so they must cross. In our numerical explorations across a wide range of parameter values, we consistently find a unique solution to equation (1). While we cannot provide a general analytical proof of uniqueness, our simulation approach involves verifying that the identified critical point is indeed a global maximizer of the profit function by checking that no profitable deviations exist. Finally, we should re-emphasize that (1) is a necessary condition, but not a sufficient one, for $P_1^D = P_2^D = P_n$ to be a Nash equilibrium. That said, once we find the solution to (1), we can verify numerically that $P_1^D = P_n$ is indeed a global best-response to $P_2^D = P_n$.

Collusive Profits

Aside from the competitive price level, we also consider the outcome if the two platforms were to collude. First, suppose the two platforms colluded by setting identical prices $P_1^D = P_2^D = P_c$ at the joint-profit-maximizing level.

Proposition 2. *In the price-setting model, joint symmetric profits are maximized at*

$$P_c = \frac{2\theta + \gamma M}{2\theta(\theta + \gamma M)}$$

which gives each firm collusive profits of

$$\Pi^c = \frac{\gamma M}{8\theta(\theta + \gamma M)}$$

To see when this maximal level of profits can be sustained in a repeated game setting via a grim-trigger (Nash reversion) strategy, we need to check whether

$$\frac{1}{1-\delta}\Pi_i(P_c, P_c) \geq \max_P \Pi_i(P, P_c) + \frac{\delta}{1-\delta}\Pi_i(P_n, P_n)$$

where P_c refers to the collusive price $\frac{2\theta+\gamma M}{2\theta(\theta+\gamma M)}$ and P_n refers to the Nash price. Rearranging, Π^c can be sustained whenever

$$\delta \geq \delta^* \equiv \frac{\max_P \Pi_i(P, P_c) - \Pi_i(P_c, P_c)}{\max_P \Pi_i(P, P_c) - \Pi_i(P_n, P_n)}.$$

For $\delta < \delta^*$, we can also calculate the highest level of collusive profits sustainable, as the solution to

$$\max_{\tilde{P}} \Pi_i(\tilde{P}, \tilde{P}) \qquad \text{subject to} \qquad \delta \quad \geq \quad \frac{\max_P \Pi_i(P, \tilde{P}) - \Pi_i(\tilde{P}, \tilde{P})}{\max_P \Pi_i(P, \tilde{P}) - \Pi_i(P_n, P_n)}.$$

This latter problem doesn't admit a closed-form solution for $\delta < \delta^*$, but can easily be solved numerically, allowing us to find the maximal prices that can be sustained in a grim-trigger equilibrium.

4 Analyzing The Fee-Setting Model

Next, we consider the peer-to-peer model of platform competition — firms set fee levels f_i, the fraction of buyer revenue retained by the platform, and both the buyer price P_i^D and the seller price P_i^S at each platform are determined by market-clearing and the restriction that $P_i^S = (1 - f_i)P_i^D$. Again, we work backwards from the end.

4.1 Stage 3: Price Determination and Trade

Suppose that platform i has set fee level f_i, and that measures N_i of buyers and M_i of sellers have chosen platform i. Market clearing now requires

$$N_i(1 - \theta P_i^D) \quad = \quad M_i \gamma P_i^S \quad = \quad M_i \gamma (1 - f_i) P_i^D \quad \to \quad P_i^D \quad = \quad \frac{N_i}{N_i \theta + M_i \gamma (1 - f_i)}$$

and with it $P_i^S = \frac{N_i(1-f_i)}{N_i\theta + M_i\gamma(1-f_i)}$. Gross buyer surplus is $\frac{1}{2\theta}\left(1 - \frac{N_i\theta}{N_i\theta + M_i\gamma(1-f_i)}\right)^2$ (before travel costs and U); gross producer surplus is $\frac{1}{2}\gamma\left(\frac{N_i(1-f_i)}{N_i\theta + M_i\gamma(1-f_i)}\right)^2$; and the platform's revenue is

$$\Pi_i \quad = \quad f_i Q_i P_i^D \quad = \quad f_i (M_i \gamma P_i^S) P_i^D \quad = \quad f_i M_i \gamma \frac{N_i(1 - f_i)}{N_i\theta + M_i\gamma(1 - f_i)} \frac{N_i}{N_i\theta + M_i\gamma(1 - f_i)}.$$

4.2 Stage 2: Buyers and Sellers Choose Platforms

Once the two platforms have chosen f_1 and f_2, the buyers and sellers each select the platform that will give them greater surplus (given correct beliefs about what the rest of the market is doing), determining N_i and M_i. This time, however, both P_i^D and P_i^S depend on beliefs about how other buyers and sellers are choosing, and so the determination of α and β is a bit more complicated.

If a measure $N_1 = \alpha$ of buyers and $M_1 = \beta M$ of sellers choose platform 1 and the rest choose platform 2, then a buyer at location x prefers platform 1 if

$$\begin{aligned} & \frac{1}{2\theta}\left(1 - \frac{(1-\alpha)\theta}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2 - t(1-x) + U \\ \leq \quad & \frac{1}{2\theta}\left(1 - \frac{\alpha\theta}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - tx + U \end{aligned}$$

or equivalently

$$x \quad \leq \quad \frac{1}{2} + \frac{1}{4t\theta}\left[\left(\frac{\beta M\gamma(1-f_1)}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - \left(\frac{(1-\beta)M\gamma(1-f_2)}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2\right].$$

Similarly, a seller at location x prefers platform 1 if

$$\frac{\gamma}{2}\left(\frac{(1-\alpha)(1-f_2)}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2 - T(1-x) + U' \quad \leq \quad \frac{\gamma}{2}\left(\frac{\alpha(1-f_1)}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - Tx + U'$$

or

$$x \quad \leq \quad \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha(1-f_1)}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - \left(\frac{(1-\alpha)(1-f_2)}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2\right].$$

As a result, given (f_1, f_2), interior market shares $(\alpha, \beta) \in (0,1)$ are an equilibrium if and only if they satisfy the system

$$\begin{aligned}
\alpha \quad &= \quad \frac{1}{2} + \frac{1}{4t\theta}\left[\left(\frac{\beta M\gamma(1-f_1)}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - \left(\frac{(1-\beta)M\gamma(1-f_2)}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2\right] \\
\beta \quad &= \quad \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha(1-f_1)}{\alpha\theta + \beta M\gamma(1-f_1)}\right)^2 - \left(\frac{(1-\alpha)(1-f_2)}{(1-\alpha)\theta + (1-\beta)M\gamma(1-f_2)}\right)^2\right].
\end{aligned}$$

Once again, without a formal proof, we can observe numerically that given (f_1, f_2), this system typically has a unique interior solution if it has one at all.[9] When there is an interior solution - an equilibrium where both platforms are "active" - we assume that it is the equilibrium played. When there is no interior solution (which requires $f_1 \neq f_2$), we assume that the equilibrium played is the unique one where all sellers are at the cheaper platform.[10]

4.3 Stage 1: Platforms Set Fee Levels

Symmetric Nash Equilibrium

Once again, we can find a computationally tractable necessary condition for symmetric equilibrium:

Proposition 3. *If $f_1 = f_2 = f$ is a symmetric Nash equilibrium of the fee-setting game, then f must satisfy*

$$0 = \underbrace{\frac{1}{f} - \frac{1}{1-f}}_{I} - \underbrace{\begin{bmatrix} \frac{4M\gamma(1-f)}{\theta + M\gamma(1-f)} & \frac{2(\theta - M\gamma(1-f))}{\theta + M\gamma(1-f)} \end{bmatrix} Q^{-1} \begin{bmatrix} M^2\gamma^2(1-f) \\ \theta\gamma(1-f) \end{bmatrix}}_{II} + \underbrace{\frac{2M\gamma}{\theta + M\gamma(1-f)}}_{III}, \quad (2)$$

[9]For f_1 and f_2 too far from each other, there are only corner solutions – equilibria where all sellers are at the same platform.

[10]Depending on the value of t, some buyers may still choose the "empty" platform.

where the matrix

$$Q = \begin{bmatrix} 4M^2\gamma^2(1-f)^2 + 2t(\theta + M\gamma(1-f))^3 & -4M^2\gamma^2(1-f)^2 \\ -4M\gamma^2(1-f)^3 & 4M\gamma^2(1-f)^3 + 2T(\theta + M\gamma(1-f))^3 \end{bmatrix}$$

is invertible.

This is a necessary but not sufficient condition for $f_1 = f_2 = f$ to be a Nash equilibrium. For any f satisfying this expression, we can verify numerically that it is indeed an equilibrium.

Equation (2) also implies that a solution always exists. As $f \to 0$, I goes to infinity while all other terms remain bounded, so the expression is strictly positive. As $f \to 1$, $Q \to \begin{bmatrix} 2t\theta^3 & 0 \\ 0 & 2T\theta^3 \end{bmatrix}$ and $\begin{bmatrix} M^2\gamma^2(1-f) \\ \theta\gamma(1-f) \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so II vanishes. Since I goes to $-\infty$ and III is bounded, the whole expression is strictly negative for f close to 1. By continuity, it crosses zero somewhere.

Collusion

Although it's not immediately obvious, two platforms colluding in the fee-setting game would get the same outcome as two platforms colluding in the price-setting game:

Proposition 4. *In the fee-setting model, joint symmetric profits are maximized at a fee level of*

$$f = \frac{\theta + M\gamma}{2\theta + M\gamma}$$

which leads to the same prices and the same collusive profits

$$\Pi^C = \frac{\gamma M}{8\theta(\theta + \gamma M)}$$

as in the price-setting model.

While the collusive profit level Π^c is the same across the two models, δ^* will be different, since both the one-period deviation profits and the Nash profits are different. For $\delta < \delta^*$, maximal collusive profits, defined by the same constrained optimization problem as before, will be different as well.

5 The Effect of Promoting Platform Switching: Results and Discussion

For our numerical simulations, we employ a computational approach that (1) solves for equilibrium prices/fees under both Nash competition and collusion for a given set of parameters, (2) verifies that these solutions satisfy first-order conditions, and (3) confirms that no profitable deviations exist by checking profits over the full strategy space. Our benchmark parameterization uses $M = 0.5$,

$\theta = 0.1$, and $\gamma = 1$, though as demonstrated in Section 5.4, our key insights hold across a wide range of parameter values.

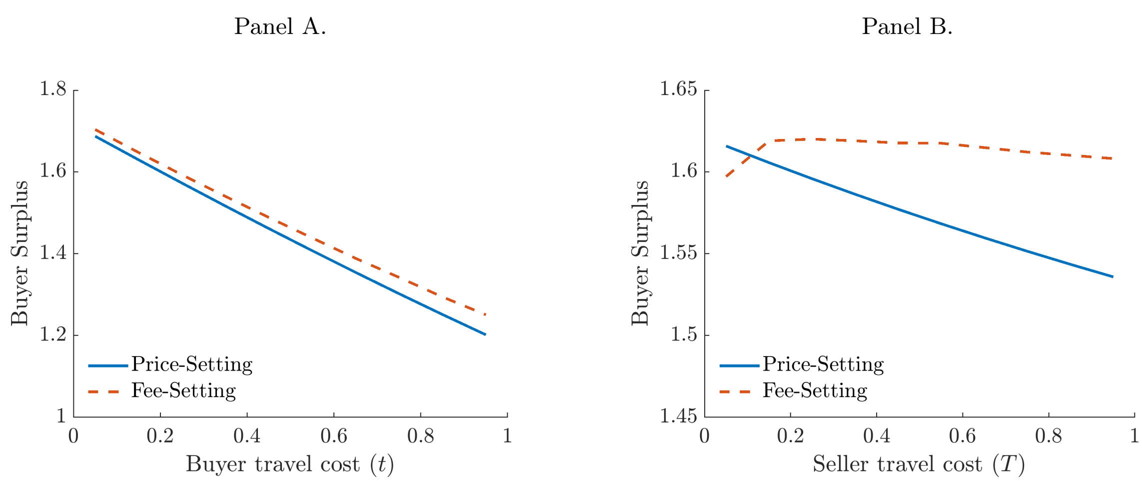
5.1 Comparative Statics of Buyer Surplus – Nash Equilibrium

The results in Propositions 1 and 3 allow us, for a given set of primitives, to numerically calculate the prices or fee levels set by the two platforms in symmetric Nash equilibrium, and from those, equilibrium payoffs. This allows us to see, at least in particular cases, how equilibrium payoffs change with environmental parameters.

To foster competition between platforms, regulators have considered policies that promote users' ability to switch between and access multiple platforms. In our model, such an intervention can be represented as a reduction in travel costs for buyers or sellers. For either buyers or sellers, as their travel cost approaches 0, switching between platforms becomes easier and they increasingly gravitate to whichever platform offers them a more profitable environment in equilibrium. We first analyze how this reduction in travel costs affects buyer surplus — a key regulatory concern — before extending the analysis to seller surplus, platform profits, and total welfare.

Figure 1 shows how equilibrium buyer surplus responds to changes in travel cost, for a particular parameterization of our two models. In Panel A, $T = 0.2$, and t varies from 0 to 1; in Panel B, $t = 0.2$, and T varies from 0 to 1. In both panels, the solid curve shows buyer surplus in the price-setting model, and the dashed curve shows buyer surplus in the fee-setting model.

FIGURE 1: Nash Equilibrium Buyer Surplus as Travel Costs Change



The figure plots Nash equilibrium buyer surplus for both the price- and fee-setting models. Panel A fixes seller travel costs at $T = 0.2$ and varies buyer travel costs t. Panel B fixes buyer travel costs at $t = 0.2$ and varies seller travel costs T. Both use parameter values $M = 0.5$, $\theta = 0.1$, and $\gamma = 1$.

Panel A shows that buyer surplus rises as buyers' travel cost falls in both the price- and fee-setting models. This happens for two reasons. First, travel costs are mechanically subtracted from buyers' surplus. Second, by making buyers more willing to switch platforms for a better "deal," reducing t leads to lower equilibrium prices or fees set by the platform under both models. Therefore greater surplus on each platform can be divided between the buyers and sellers. In symmetric equilibrium, the average buyer incurs travel costs of $\frac{1}{4}t$; since equilibrium buyer surplus under both models rises from about 1.2 to about 1.7 as t drops from 1 to 0, the two effects are roughly the same size.

Panel B shows that in the price-setting model, buyer surplus also rises as *seller* travel costs fall. The two panels have different scales on the y axis; the effect is not nearly as strong as the effect of buyers' own travel costs, due to the lack of a direct effect; in this case, the rise in buyer surplus comes only from the platforms' reduction in prices as T falls. As T falls, buyers remain equally responsive to a change in price, but sellers become more responsive to a change in the location of buyers; this increases the benefit to a platform of lowering its buyer-facing price as T falls.

However, buyer surplus changes very little in response to a drop in seller travel costs under the fee-setting model, first rising as T falls, but then falling as T gets even smaller. This follows the pattern in equilibrium fee levels: platform fee levels respond very little to changes in seller travel costs, first falling very slowly with a decrease in T but then rising as T falls once T is very low. Since platform fees scale with buyer prices, platforms can tolerate losing some sellers - the resulting higher market-clearing prices offset the reduced transaction volume, meaning a smaller market size (T) may not substantially lower equilibrium fees. This non-monotonicity in the fee-setting model stems from the interplay between two opposing forces. As seller travel costs decrease, sellers become more responsive to differences between platforms, which initially incentivizes platforms to compete more aggressively on fees, benefiting buyers. However, when seller travel costs become very low, platforms can more easily attract sellers even with higher fees, allowing them to extract more surplus from the value chain as a whole, potentially harming buyers.

The upshot, then, is that if the two platforms play the static Nash equilibrium, buyers benefit from policies that make it easier for them to switch platforms under either model; and also benefit from policies that make it easier for sellers to switch platforms under the price-setting model.

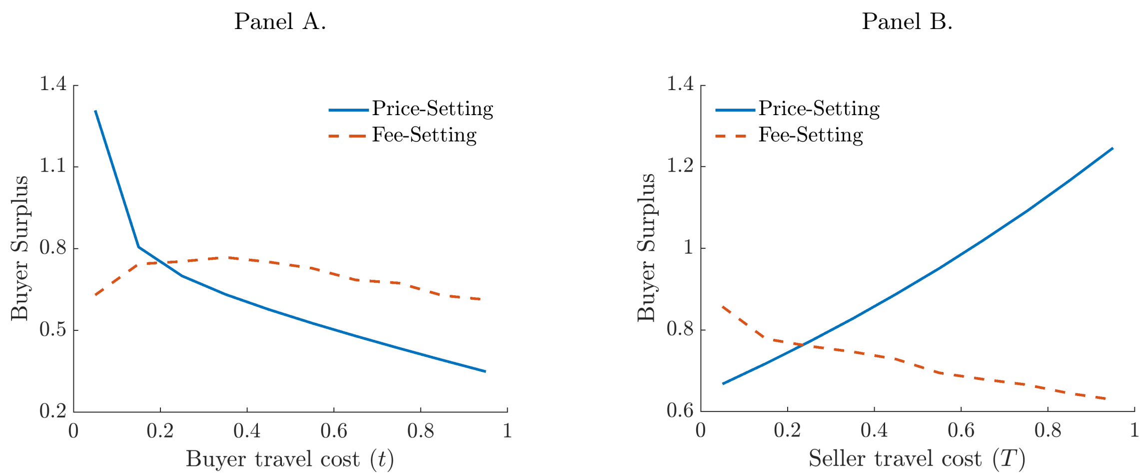
5.2 Comparative Statics of Buyer Surplus – Collusive Platforms

The results in Propositions 2 and 4 establish the maximal profits platforms could achieve in either model at the joint-profit-maximizing price or fee level. For sufficiently high discount rates δ, these profits are sustainable in the infinitely repeated game through threats of Nash reversion after deviations. When δ falls below a critical value, while maximum profits are no longer sustainable, we can calculate the highest achievable profits in a grim-trigger equilibrium by analyzing three factors: collusive profits at a given fee level, profits from the optimal deviation, and Nash profits. This allows us to determine buyer surplus and welfare measures at the corresponding price or fee levels, addressing a key question: if platforms are colluding, will regulatory interventions have their intended effect, or might they work differently—possibly even opposite to regulators' intentions?

The sustainable collusive prices or fees respond to market parameters in complex ways because sustainability depends on both the short-term gains from deviation and the severity of Nash punishment. Since these components can respond differently to changes in market primitives, the comparative statistics become difficult to predict, but can be studied in simulations. Figure 2 shows buyer surplus, for the same environment as considered above, under the added assumption that the platforms have discount factor $\delta = 0.5$ and collude to the maximum extent possible in a grim-trigger equilibrium.

Panel A shows differing effects of reduced buyer travel costs under collusion. In the price-setting model, buyer surplus rises consistently as t falls. However, in the fee-setting model, buyer surplus first increases but then decreases as t approaches zero. This non-monotonic pattern emerges because falling travel costs reduce Nash profits while increasing deviation profits, leading to relatively stable but non-monotonic maximum collusive prices.

FIGURE 2: Buyer Surplus under Platform Collusion as Travel Costs Change



The figure plots buyer surplus for both the price- and fee-setting models under the assumption of maximal sustainable collusion with $\delta = 0.5$. Panel A fixes seller travel costs at $T = 0.2$ and varies buyer travel costs t. Panel B fixes buyer travel costs at $t = 0.2$ and varies seller travel costs T. Both use $M = 0.5$, $\theta = 0.1$, and $\gamma = 1$.

Panel B reveals contrasting effects of reduced seller travel costs (T). Under fee-setting, buyer surplus increases monotonically as T falls. However, under price-setting, buyer surplus declines sharply. This decline occurs because lower T reduces Nash prices and profits, making collusion more sustainable through harsher punishment for deviation. Consequently, platforms can maintain higher collusive prices. This suggests that policies facilitating seller platform switching may benefit buyers when platforms compete but *harm* them when platforms collude, particularly in price-setting scenarios.

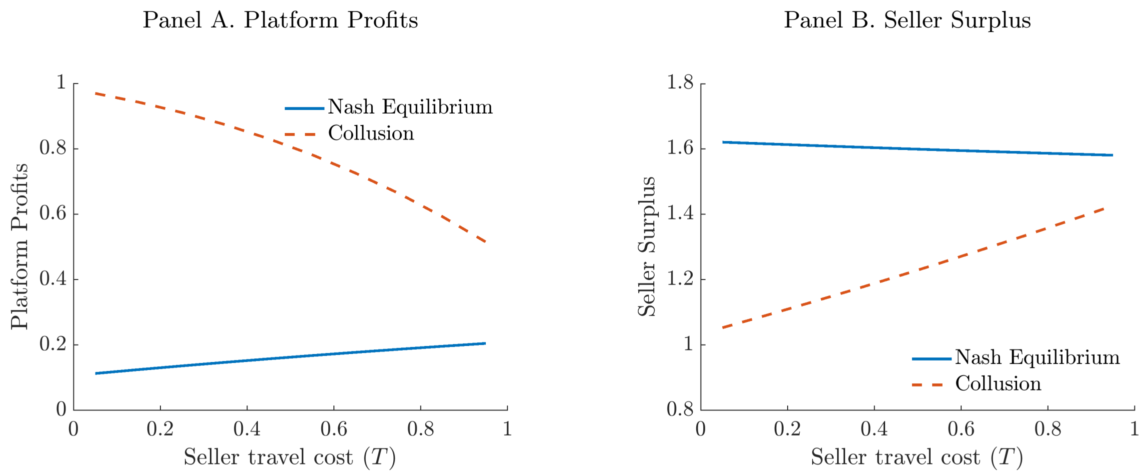
5.3 Other Comparative Statics

We can similarly look at comparative statics with respect to the other parties' payoffs. We focus on the price-setting model. As discussed above, when platforms compete (static Nash), platform profits decrease as seller travel costs decrease, as "closer competition" between platforms leads to lower platform prices. However, when the platforms are colluding, the same reduction in seller travel costs now *increases* platform profits, by increasing the price they can sustain in a collusive equilibrium. (While the payoff from deviating from the collusive price increases, the decrease in

Nash profits offsets this, and a higher price can be sustained.) These two results are shown in Panel A of Figure 3.

Panel B shows the effect of seller travel costs on *seller* surplus. When the platforms are competing (static Nash), a reduction in T increases seller surplus, as we would expect. But when platforms are colluding, the reduction in T now reduces seller surplus: the higher price sustained by platforms in the collusive equilibrium reduces volumes enough to hurt sellers even more than they benefit from the direct reduction in their travel costs. Thus, when the platforms are actually colluding, a well-intentioned policy intervention that reduces barriers to sellers platform switching can actually benefit the platforms, at the expense of *both buyers and sellers* – exactly the opposite of the intended effect.

FIGURE 3: Other Effects of Seller Travel Costs in Price-Setting Model

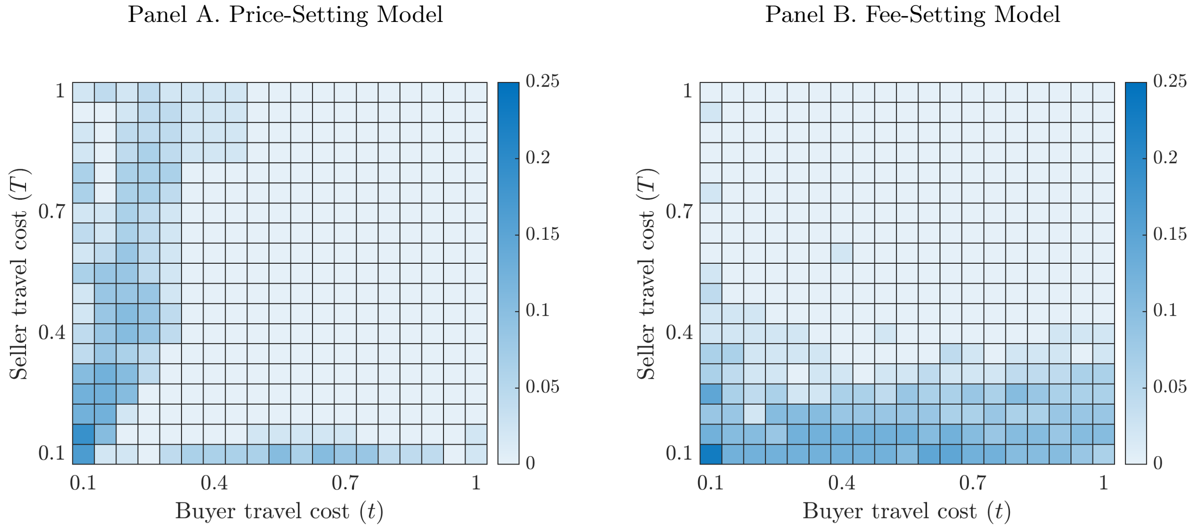


The figure compares platform profits and seller surplus under Nash equilibrium and maximal sustainable collusion with $\delta = 0.5$. Both panels fix buyer travel costs at $t = 0.2$ and vary seller travel costs T; both use $M = 0.5$, $\theta = 0.1$, and $\gamma = 1$.

5.4 Beyond One Example

Finally, we argue that while not generic, the reversals highlighted above are not overly cherry-picked. To support this claim, we consider a wide range of different parametrizations of both models under collusion, varying the ratio of buyers to sellers, buyers' and sellers' travel costs, and the slope parameters on each buyer's demand and each seller's supply. For each level of buyer and seller travel costs, we consider the fraction of parametrizations under which buyer surplus *falls* when buyers' and sellers' travel costs are reduced slightly. Figure 4 presents these results graphically as a "heat map": Panel A shows the results for the price-setting model, and Panel B for the fee-setting model. While buyer surplus is still more likely to rise than fall when both buyer and seller travel costs fall, Figure 4 shows that cases where buyer surplus falls are relatively common when buyer travel cost is already low under the price-setting model, and when seller travel cost is already low under the fee-setting model; when both sides' travel costs are already low, this reversal (buyers being hurt as platform switching becomes easier) occur in about 20% of parametrizations tested.

FIGURE 4: Prevalence of Buyer Surplus Falling when T and t Both Fall



For each value of t and T, we vary $M \in \{0.5, 1, 2\}$, $\theta \in \{0.1, 0.5, 1, 2\}$, and $\gamma \in \{0.1, 0.5, 1, 2\}$; a darker cell represents a higher fraction of these parameterizations where buyer surplus falls when buyer and seller travel costs are both reduced by 0.05. Panel A presents the results for the price-setting model, Panel B for the fee-setting model.

6 Conclusion

In this paper, we have examined the theoretical implications of policies that aim to facilitate cross-platform participation in digital markets. By developing a model that captures key aspects of platform competition—horizontal differentiation, transaction-by-transaction platform choice, and different pricing mechanisms—we have shown that the effects of reducing barriers to platform switching vary significantly depending on specific features of the market. While policies that reduce transportation costs for buyers or sellers generally increase consumer welfare in competitive settings, these same policies can have ambiguous or even negative welfare effects when platforms collude. This reversal is particularly pronounced in the price-setting model, where reducing seller travel costs under collusion leads to dramatically lower buyer and seller surplus, in stark contrast to the competitive case. Our findings suggest that policymakers should exercise caution when implementing interventions designed to facilitate platform switching: effective policy design may require not only assessing the likelihood of platform collusion but also understanding the specific business models employed by platforms in the targeted market.

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Appendix – Proofs

Some mechanical calculations omitted here are given in full in a separate supplemental appendix.

A.1 Proof of Proposition 1

We want $P_1^D = P$ to be a local best-response to $P_2^D = P$. Differentiating Π_1 with respect to P_1^D gives

$$\begin{aligned}\frac{d\Pi_1}{dP_1^D} &= \alpha'\left(P_1^D - \theta(P_1^D)^2 - \frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)^2\right)\\ &\quad +\alpha\left(1-2\theta P_1^D - \frac{\alpha'}{\beta\gamma M}(1-\theta P_1^D)^2 + \frac{\alpha\beta'}{\beta^2\gamma M}(1-\theta P_1^D)^2 + 2\theta\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)\end{aligned}$$

where α' and β' refer to $\frac{\partial\alpha}{\partial P_1^D}$ and $\frac{\partial\beta}{\partial P_1^D}$, respectively. Since we're looking for symmetric equilibrium, we're only concerned with the value of $d\Pi_1/dP_1^D$ at $P_1^D = P_2^D$ and therefore $\alpha = \beta = \frac{1}{2}$, in which case this simplifies to

$$\begin{aligned}\left.\frac{d\Pi_1}{dP_1^D}\right|_{P_1^D=P_2^D=P^D} &= \alpha'\left(P^D - \theta(P^D)^2 - \frac{1}{\gamma M}(1-\theta P^D)^2\right)\\ &\quad +\frac{1}{2}\left(1-2\theta P^D - \frac{\alpha'}{\frac{1}{2}\gamma M}(1-\theta P^D)^2 + \frac{\beta'}{\frac{1}{2}\gamma M}(1-\theta P^D)^2 + 2\theta\frac{1}{\gamma M}(1-\theta P^D)\right)\end{aligned}$$

Differentiating α gives

$$\alpha = \frac{1}{2} + \frac{1}{4t\theta}\left[(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2\right] \longrightarrow \alpha' = -\frac{2\theta}{4t\theta}(1-\theta P_1^D) = -\frac{1}{2t}(1-\theta P_1^D)$$

Calculating β' takes more work: if we totally differentiate with respect to P_1^D,

$$\begin{aligned}\beta' &= \frac{\gamma}{4T}\left[2\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)\left(\frac{\alpha'}{\beta\gamma M}(1-\theta P_1^D) - \frac{\alpha\beta'}{\beta^2\gamma M}(1-\theta P_1^D) - \theta\frac{\alpha}{\beta\gamma M}\right)\right.\\ &\quad \left. -2\left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)\left(\frac{-\alpha'}{(1-\beta)\gamma M}(1-\theta P_2^D) + \frac{(1-\alpha)\beta'}{(1-\beta)^2\gamma M}(1-\theta P_2^D)\right)\right]\end{aligned}$$

Plugging in $P_1^D = P_2^D$ and $\alpha = \beta = \frac{1}{2}$ and simplifying gives

$$\beta'\big|_{P_1^D = P_2^D} = \frac{4\alpha'(1-\theta P^D)^2 - \theta(1-\theta P^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2}$$

Plugging α' and β' into $d\Pi_1/dP_1^D$ at $P_1^D = P_2^D = P^D$ and simplifying gives

$$\frac{d\Pi_1}{dP_1^D}\bigg|_{P_1^D = P_2^D = P^D} = \left(-\frac{1}{2t}(1-\theta P^D)\right)\left(P^D(1-\theta(P^D)) - \frac{1}{\gamma M}(1-\theta P^D)^2\right)$$
$$+\frac{1}{2} - \theta P^D + \frac{1}{\gamma M}\frac{\left(\frac{T\gamma M^2}{t} - \theta\right)(1-\theta P_1^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2}(1-\theta P^D)^2 + \theta\frac{1}{\gamma M}(1-\theta P^D)$$

Setting this equal to 0 and multiplying by $2t\gamma M$ gives

$$t\gamma M\theta P^D = -\gamma M P^D(1-\theta P^D)^2 + (1-\theta P^D)^3 + t\gamma M(1-\theta P^D) + 2t\theta(1-\theta P^D)$$
$$+\frac{T\gamma M^2 - t\theta}{T\gamma M^2 + 2(1-\theta P^D)^2}(1-\theta P^D)^3$$

proving the proposition. □

A.2 Proof of Proposition 2

Prices $P_1^D = P_2^D = P^D$ lead to equal market shares $\alpha = \beta = \frac{1}{2}$ and, by market clearing, $P^S = \frac{N_i}{M_i\gamma}(1-\theta P^D) = \frac{1}{\gamma M}(1-\theta P^D)$ at each platform. Each platform would then earn

$$\Pi_i(\cdot) = Q_i(P^D - P^S) = \frac{1}{2}(1-\theta P^D)\left(P^D - \frac{1}{\gamma M}(1-\theta P^D)\right)$$

Maximizing this gives

$$P^D = \frac{2\theta + \gamma M}{2\theta(\gamma M + \theta)}$$

Plugging this into the expression for revenue and simpifying gives

$$\Pi^c = \frac{1}{2\gamma M}\left(\frac{\gamma M}{2(\gamma M + \theta)}\right)\left(\frac{\gamma M}{2\theta}\right) = \frac{\gamma M}{8\theta(\gamma M + \theta)}$$

completing the proof. □

A.3 Proof of Proposition 3

We again begin with platform 1's revenue, which in the fee-setting game is

$$\Pi_1 \quad = \quad f_1 \beta M \gamma \frac{\alpha^2 (1-f_1)}{(\alpha\theta + \beta M \gamma (1-f_1))^2}$$

If we take the log and then differentiate with respect to f_1,

$$\frac{d \log \Pi_1}{d f_1} \quad = \quad \frac{1}{f_1} + \frac{\beta'}{\beta} + 2\frac{\alpha'}{\alpha} - \frac{1}{1-f_1} - 2\frac{\alpha'\theta + \beta' M\gamma(1-f_1) - \beta M \gamma}{\alpha\theta + \beta M\gamma(1-f_1)}$$

Totally differentiating the expression for α with respect to f_1, then plugging in $f_1 = f_2 = f$ and $\alpha = \beta = \frac{1}{2}$ and simplifying, gives

$$2t\theta\alpha' \quad = \quad \left(\frac{M\gamma(1-f)}{\theta + M\gamma(1-f)}\right)\left(-\frac{4\alpha'\theta}{\theta + M\gamma(1-f)} + \frac{\theta(4\alpha'\theta + 4\beta' M\gamma(1-f) - M\gamma)}{(\theta + M\gamma(1-f))^2}\right)$$

where now α' and β' refer to $\partial\alpha/\partial f_1$ and $\partial\beta/\partial f_1$; further manipulation lets us rewrite this as

$$M^2\gamma^2(1-f) \quad = \quad -\left[4M^2\gamma^2(1-f)^2 + 2t(\theta + M\gamma(1-f))^3\right]\alpha' + \left[4M^2\gamma^2(1-f)^2\right]\beta' \qquad (3)$$

Similarly, totally differentiating the expression for β, plugging in $f_1 = f_2 = f$ and $\alpha = \beta = \frac{1}{2}$, and simplifying leads to

$$\beta' \quad = \quad \frac{\gamma}{2T}\left(\frac{1-f}{\theta + M\gamma(1-f)}\right)\left(\frac{4\alpha'(1-f) - 1}{\theta + M\gamma(1-f)} - \frac{(1-f)(4\alpha'\theta + 4\beta' M\gamma(1-f) - M\gamma)}{(\theta + M\gamma(1-f))^2}\right)$$

which simplifies further to

$$\theta\gamma(1-f) \quad = \quad \left[4M\gamma^2(1-f)^3\right]\alpha' - \left[4M\gamma^2(1-f)^3 + 2T(\theta + M\gamma(1-f))^3\right]\beta' \qquad (4)$$

Flipping signs, we can write equations (3) and (4) as

$$Q\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} \quad = \quad \begin{bmatrix} -M^2\gamma^2(1-f) \\ -\theta\gamma(1-f) \end{bmatrix}$$

where

$$Q \quad = \quad \begin{bmatrix} 4M^2\gamma^2(1-f)^2 + 2t(\theta + M\gamma(1-f))^3 & -4M^2\gamma^2(1-f)^2 \\ & \\ -4M\gamma^2(1-f)^3 & 4M\gamma^2(1-f)^3 + 2T(\theta + M\gamma(1-f))^3 \end{bmatrix}$$

Letting $z = (\theta + M\gamma(1-f))$, note that

$$|Q| = \left(4M^2\gamma^2(1-f)^2 + 2tz^3\right)\left(4M\gamma^2(1-f)^3 + 2Tz^3\right) - \left(-4M^2\gamma^2(1-f)^2\right)\left(-4M\gamma^2(1-f)^3\right)$$

$$= 8M^2T\gamma^2(1-f)^2z^3 + 8Mt\gamma^2(1-f)^3z^3 + 4tTz^6 \quad > \quad 0$$

so Q is invertible, and therefore

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = Q^{-1}\begin{bmatrix} -M^2\gamma^2(1-f) \\ -\theta\gamma(1-f) \end{bmatrix}$$

Returning to our first-order condition from before, plugging in $f_1 = f_2 = f$ and $\alpha = \beta = \frac{1}{2}$ and simplifying gives

$$0 = \frac{1}{f} - \frac{1}{1-f} + \begin{bmatrix} \frac{4M\gamma(1-f)}{\theta+M\gamma(1-f)} & \frac{2\theta-2M\gamma(1-f)}{\theta+M\gamma(1-f)} \end{bmatrix}\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} + \frac{2M\gamma}{\theta+M\gamma(1-f)}$$

$$= \frac{1}{f} - \frac{1}{1-f} + \begin{bmatrix} \frac{4M\gamma(1-f)}{\theta+M\gamma(1-f)} & \frac{2\theta-2M\gamma(1-f)}{\theta+M\gamma(1-f)} \end{bmatrix} Q^{-1}\begin{bmatrix} -M^2\gamma^2(1-f) \\ -\theta\gamma(1-f) \end{bmatrix} + \frac{2M\gamma}{\theta+M\gamma(1-f)}$$

concluding the proof. □

A.4 Proof of Proposition 4

If the two platforms colluded by setting identical fee levels $f_1 = f_2 = f$, this would lead to equal market shares $\alpha = \beta = \frac{1}{2}$ and a buyer price

$$P^D = \frac{N_i}{N_i\theta + M_i\gamma(1-f_i)} = \frac{1}{\theta + M\gamma(1-f)}$$

at each platform. Demand at each platform would be

$$Q_i = M_i\gamma P_i^S = \frac{1}{2}M\gamma\frac{1-f}{\theta+M\gamma(1-f)}$$

giving each platform revenue of

$$\Pi_i = Q_i f_i P_i^D = \frac{1}{2}\frac{M\gamma f(1-f)}{(\theta+M\gamma(1-f))^2}$$

Taking the log and dropping the additive constants, maximizing

$$\log f + \log(1-f) - 2\log(\theta + M\gamma(1-f))$$

gives the first-order condition

$$\frac{1}{f} - \frac{1}{1-f} + \frac{2M\gamma}{\theta + M\gamma(1-f)} \quad = \quad 0$$

which simplifies to

$$f \quad = \quad \frac{\theta + M\gamma}{2\theta + M\gamma}$$

Along with equal market shares, this implies a buyer price

$$P^D \quad = \quad \frac{1}{\theta + M\gamma\left(1 - \frac{\theta+M\gamma}{2\theta+M\gamma}\right)} \quad = \quad \frac{2\theta + M\gamma}{\theta(2\theta + M\gamma) + M\gamma\left(2\theta + M\gamma - (\theta + M\gamma)\right)} \quad = \quad \frac{2\theta + \gamma M}{2\theta(\theta + \gamma M)}$$

and seller price

$$P^S \quad = \quad (1-f)P^D \quad = \quad \left(1 - \frac{\theta + M\gamma}{2\theta + M\gamma}\right)\frac{2\theta + \gamma M}{2\theta(\theta + \gamma M)} \quad = \quad \frac{1}{2(\theta + \gamma M)}$$

which are the same as the collusive outcome in the price-setting game, and therefore lead to the same level of profits. □

Supplemental Appendix to "Platform Switching Policies under Collusion"

S.1 Proof of Proposition 1

We focus on finding a condition on P for $P_1^D = P$ to be a local best-response to $P_2^D = P$. Differentiating Π_1 with respect to P_1^D gives

$$\begin{aligned}
\Pi_1 &= \alpha\left(P_1^D - \theta(P_1^D)^2 - \frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)^2\right) \\
&\downarrow \\
\frac{d\Pi_1}{dP_1^D} &= \alpha'\left(P_1^D - \theta(P_1^D)^2 - \frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)^2\right) \\
&\quad +\alpha\left(1 - 2\theta P_1^D - \frac{\alpha'}{\beta\gamma M}(1-\theta P_1^D)^2 + \frac{\alpha\beta'}{\beta^2\gamma M}(1-\theta P_1^D)^2 + 2\theta\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)
\end{aligned}$$

where α' and β' refer to $\frac{\partial\alpha}{\partial P_1^D}$ and $\frac{\partial\beta}{\partial P_1^D}$, respectively. Since we're looking for symmetric equilibrium, we're only concerned with the value of $d\Pi_1/dP_1^D$ at $P_1^D = P_2^D$ and therefore $\alpha = \beta = \frac{1}{2}$, in which case this simplifies to

$$\begin{aligned}
\left.\frac{d\Pi_1}{dP_1^D}\right|_{P_1^D=P_2^D=P^D} &= \alpha'\left(P^D - \theta(P^D)^2 - \frac{1}{\gamma M}(1-\theta P^D)^2\right) \\
&\quad +\frac{1}{2}\left(1 - 2\theta P^D - \frac{\alpha'}{\frac{1}{2}\gamma M}(1-\theta P^D)^2 + \frac{\beta'}{\frac{1}{2}\gamma M}(1-\theta P^D)^2 + 2\theta\frac{1}{\gamma M}(1-\theta P^D)\right)
\end{aligned}$$

Differentiating α gives

$$\alpha = \frac{1}{2} + \frac{1}{4t\theta}\left[(1-\theta P_1^D)^2 - (1-\theta P_2^D)^2\right] \longrightarrow \alpha' = -\frac{2\theta}{4t\theta}(1-\theta P_1^D) = -\frac{1}{2t}(1-\theta P_1^D)$$

Calculating β' takes more work: if we totally differentiate with respect to P_1^D,

$$\begin{aligned}
\beta &= \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)^2 - \left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)^2\right] \\
&\downarrow \\
\beta' &= \frac{\gamma}{4T}\left[2\left(\frac{\alpha}{\beta\gamma M}(1-\theta P_1^D)\right)\left(\frac{\alpha'}{\beta\gamma M}(1-\theta P_1^D) - \frac{\alpha\beta'}{\beta^2\gamma M}(1-\theta P_1^D) - \theta\frac{\alpha}{\beta\gamma M}\right)\right. \\
&\quad \left. -2\left(\frac{1-\alpha}{(1-\beta)\gamma M}(1-\theta P_2^D)\right)\left(\frac{-\alpha'}{(1-\beta)\gamma M}(1-\theta P_2^D) + \frac{(1-\alpha)\beta'}{(1-\beta)^2\gamma M}(1-\theta P_2^D)\right)\right]
\end{aligned}$$

Plugging in $P_1^D = P_2^D$ and $\alpha = \beta = \frac{1}{2}$, this simplifies to

$$\begin{aligned}
\beta' &= \frac{\gamma}{4T}\left[2\left(\frac{1}{\gamma M}(1-\theta P^D)\right)\left(2\frac{\alpha'}{\frac{1}{2}\gamma M}(1-\theta P^D) - 2\frac{\beta'}{\frac{1}{2}\gamma M}(1-\theta P^D) - \theta\frac{1}{\gamma M}\right)\right] \\
2T\gamma M^2\beta' &= 4\alpha'(1-\theta P^D)^2 - 4\beta'(1-\theta P^D)^2 - \theta(1-\theta P^D) \\
\beta'|_{P_1^D=P_2^D} &= \frac{4\alpha'(1-\theta P^D)^2 - \theta(1-\theta P^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2}
\end{aligned}$$

and, since we actually care about $\beta' - \alpha'$,

$$\begin{aligned}
(\beta'-\alpha')|_{P_1^D=P_2^D} &= \frac{4\alpha'(1-\theta P^D)^2 - \theta(1-\theta P^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2} - \alpha' \\
&= \frac{-2T\gamma M^2\alpha' - \theta(1-\theta P^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2} \qquad = \frac{\left(\frac{T\gamma M^2}{t} - \theta\right)(1-\theta P_1^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2}
\end{aligned}$$

Returning to our original problem to plug in α' and β',

$$\begin{aligned}
\left.\frac{d\Pi_1}{dP_1^D}\right|_{P_1^D=P_2^D=P^D} &= \alpha'\left(P^D(1-\theta(P^D)) - \frac{1}{\gamma M}(1-\theta P^D)^2\right) \\
&\quad + \frac{1}{2} - \theta P^D + \frac{\beta'-\alpha'}{\gamma M}(1-\theta P^D)^2 + \theta\frac{1}{\gamma M}(1-\theta P^D) \\
&= \left(-\frac{1}{2t}(1-\theta P^D)\right)\left(P^D(1-\theta(P^D)) - \frac{1}{\gamma M}(1-\theta P^D)^2\right) \\
&\quad + \frac{1}{2} - \theta P^D + \frac{1}{\gamma M}\frac{\left(\frac{T\gamma M^2}{t} - \theta\right)(1-\theta P_1^D)}{2T\gamma M^2 + 4(1-\theta P^D)^2}(1-\theta P^D)^2 + \theta\frac{1}{\gamma M}(1-\theta P^D)
\end{aligned}$$

Setting this equal to 0 and multiplying by $2t\gamma M$ gives

$$\begin{aligned}
t\gamma M\theta P^D &= -\gamma M P^D(1-\theta P^D)^2 + (1-\theta P^D)^3 + t\gamma M(1-\theta P^D) + 2t\theta(1-\theta P^D) \\
&\quad + \frac{T\gamma M^2 - t\theta}{T\gamma M^2 + 2(1-\theta P^D)^2}(1-\theta P^D)^3
\end{aligned}$$

proving the proposition. □

S.2 Proof of Proposition 2

Prices $P_1^D = P_2^D = P^D$ lead to equal market shares $\alpha = \beta = \frac{1}{2}$ and, by market clearing, $P^S = \frac{N_i}{M_i \gamma}(1 - \theta P^D) = \frac{1}{\gamma M}(1 - \theta P^D)$ at each platform. Each platform would then earn

$$\begin{aligned} \Pi_i(\cdot) &= Q_i(P^D - P^S) = \frac{1}{2}(1-\theta P^D)\left(P^D - \frac{1}{\gamma M}(1-\theta P^D)\right) \\ &= \frac{1}{2\gamma M}(1-\theta P^D)\left((\gamma M + \theta)P^D - 1\right) \end{aligned}$$

Maximizing this gives first-order condition

$$\begin{aligned} 0 &= -\theta\left((\gamma M + \theta)P^D - 1\right) + (\gamma M + \theta)(1 - \theta P^D) \\ P^D\left(\theta(\gamma M + \theta) + \theta(\gamma M + \theta)\right) &= \theta + \gamma M + \theta \\ P^D &= \frac{2\theta + \gamma M}{2\theta(\gamma M + \theta)} \end{aligned}$$

Plugging this into the expression for revenue gives

$$\begin{aligned} \Pi^c &= \frac{1}{2\gamma M}\left(1 - \theta\frac{2\theta + \gamma M}{2\theta(\gamma M + \theta)}\right)\left((\gamma M + \theta)\frac{2\theta + \gamma M}{2\theta(\gamma M + \theta)} - 1\right) \\ &= \frac{1}{2\gamma M}\left(\frac{\gamma M}{2(\gamma M + \theta)}\right)\left(\frac{\gamma M}{2\theta}\right) = \frac{\gamma M}{8\theta(\gamma M + \theta)} \end{aligned}$$

completing the proof. □

S.3 Proof of Proposition 3

We again begin with platform 1's revenue, which in the fee-setting game is

$$\Pi_1 = f_1 \beta M \gamma \frac{\alpha^2(1 - f_1)}{(\alpha\theta + \beta M \gamma(1 - f_1))^2}$$

If we take the log and then differentiate with respect to f_1,

$$\begin{aligned} \log \Pi_1 &= \log f_1 + \log \beta + \log M + \log \gamma + 2\log\alpha + \log(1 - f_1) - 2\log(\alpha\theta + \beta M\gamma(1 - f_1)) \\ &\downarrow \\ \frac{d \log \Pi_1}{df_1} &= \frac{1}{f_1} + \frac{\beta'}{\beta} + 2\frac{\alpha'}{\alpha} - \frac{1}{1 - f_1} - 2\frac{\alpha'\theta + \beta' M\gamma(1 - f_1) - \beta M \gamma}{\alpha\theta + \beta M\gamma(1 - f_1)} \end{aligned}$$

Totally differentiating the expression for α with respect to f_1,

$$\begin{aligned}
\alpha &= \frac{1}{2}+\frac{1}{4t\theta}\left[\left(1-\frac{\alpha\theta}{\alpha\theta+\beta M\gamma(1-f_1)}\right)^2-\left(1-\frac{(1-\alpha)\theta}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)}\right)^2\right] \\
&\downarrow \\
\alpha' &= \frac{2}{4t\theta}\left(1-\frac{\alpha\theta}{\alpha\theta+\beta M\gamma(1-f_1)}\right)\left(-\frac{\alpha'\theta}{\alpha\theta+\beta M\gamma(1-f_1)}+\frac{\alpha\theta(\alpha'\theta+\beta' M\gamma(1-f_1)-\beta M\gamma)}{(\alpha\theta+\beta M\gamma(1-f_1))^2}\right) \\
&\quad -\frac{2}{4t\theta}\left(1-\frac{(1-\alpha)\theta}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)}\right) \\
&\qquad \left(\frac{\alpha'\theta}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)}+\frac{(1-\alpha)\theta(-\alpha'\theta-\beta' M\gamma(1-f_2))}{((1-\alpha)\theta+(1-\beta)M\gamma(1-f_2))^2}\right)
\end{aligned}$$

where now α' and β' refer to $\partial\alpha/\partial f_1$ and $\partial\beta/\partial f_1$. Plugging in $f_1=f_2=f$ and therefore $\alpha=\beta=\frac{1}{2}$, this simplifies to

$$\begin{aligned}
2t\theta\alpha' &= \left(1-\frac{\theta}{\theta+M\gamma(1-f)}\right)\left(-\frac{2\alpha'\theta}{\theta+M\gamma(1-f)}+\frac{\theta(2\alpha'\theta+2\beta' M\gamma(1-f)-M\gamma)}{(\theta+M\gamma(1-f))^2}\right) \\
&\quad -\left(1-\frac{\theta}{\theta+M\gamma(1-f)}\right)\left(\frac{2\alpha'\theta}{\theta+M\gamma(1-f)}+\frac{\theta(-2\alpha'\theta-2\beta' M\gamma(1-f))}{(\theta+M\gamma(1-f))^2}\right) \\
&= \left(\frac{M\gamma(1-f)}{\theta+M\gamma(1-f)}\right)\left(-\frac{4\alpha'\theta}{\theta+M\gamma(1-f)}+\frac{\theta(4\alpha'\theta+4\beta' M\gamma(1-f)-M\gamma)}{(\theta+M\gamma(1-f))^2}\right)
\end{aligned}$$

and, from there,

$$\begin{aligned}
(\theta+M\gamma(1-f))^3 2t\theta\alpha' &= M\gamma(1-f)\left(-4\alpha'\theta(\theta+M\gamma(1-f))+\theta(4\alpha'\theta+4\beta' M\gamma(1-f)-M\gamma)\right) \\
(\theta+M\gamma(1-f))^3 2t\theta\alpha' &= M\gamma(1-f)\left(-4\alpha'\theta M\gamma(1-f)+4\beta'\theta M\gamma(1-f)-M\gamma\theta\right) \\
(\theta+M\gamma(1-f))^3 2t\alpha' &= -4\alpha' M^2\gamma^2(1-f)(1-f)+4\beta' M^2\gamma^2(1-f)(1-f)-M^2\gamma^2(1-f) \\
M^2\gamma^2(1-f) &= -\left[4M^2\gamma^2(1-f)^2+2t(\theta+M\gamma(1-f))^3\right]\alpha'+\left[4M^2\gamma^2(1-f)^2\right]\beta'
\end{aligned}$$

Similarly, totally differentiating the expression for β gives

$$\begin{aligned}
\beta &= \frac{1}{2} + \frac{\gamma}{4T}\left[\left(\frac{\alpha(1-f_1)}{\alpha\theta+\beta M\gamma(1-f_1)}\right)^2 - \left(\frac{(1-\alpha)(1-f_2)}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)}\right)^2\right] \\
&\downarrow \\
\beta' &= \frac{2\gamma}{4T}\left(\frac{\alpha(1-f_1)}{\alpha\theta+\beta M\gamma(1-f_1)}\right)\left(\frac{\alpha'(1-f_1)-\alpha}{\alpha\theta+\beta M\gamma(1-f_1)} - \frac{\alpha(1-f_1)(\alpha'\theta+\beta' M\gamma(1-f_1)-\beta M\gamma)}{(\alpha\theta+\beta M\gamma(1-f_1))^2}\right) \\
&\quad -\frac{2\gamma}{4T}\left(\frac{(1-\alpha)(1-f_2)}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)}\right) \\
&\quad\quad \left(\frac{-\alpha'(1-f_2)}{(1-\alpha)\theta+(1-\beta)M\gamma(1-f_2)} - \frac{(1-\alpha)(1-f_2)\left(-\alpha'\theta-\beta' M\gamma(1-f_2)\right)}{((1-\alpha)\theta+(1-\beta)M\gamma(1-f_2))^2}\right)
\end{aligned}$$

Again plugging in $f_1 = f_2 = f$ and $\alpha = \beta = \frac{1}{2}$, this becomes

$$\begin{aligned}
\beta' &= \frac{\gamma}{2T}\left(\frac{1-f}{\theta+M\gamma(1-f)}\right)\left(\frac{2\alpha'(1-f)-1}{\theta+M\gamma(1-f)} - \frac{(1-f)(2\alpha'\theta+2\beta' M\gamma(1-f)-M\gamma)}{(\theta+M\gamma(1-f))^2}\right) \\
&\quad +\frac{\gamma}{2T}\left(\frac{1-f}{\theta+M\gamma(1-f)}\right)\left(\frac{2\alpha'(1-f)}{\theta+M\gamma(1-f)} - \frac{(1-f)\left(2\alpha'\theta+2\beta' M\gamma(1-f)\right)}{(\theta+M\gamma(1-f))^2}\right) \\
\beta' &= \frac{\gamma}{2T}\left(\frac{1-f}{\theta+M\gamma(1-f)}\right)\left(\frac{4\alpha'(1-f)-1}{\theta+M\gamma(1-f)} - \frac{(1-f)(4\alpha'\theta+4\beta' M\gamma(1-f)-M\gamma)}{(\theta+M\gamma(1-f))^2}\right)
\end{aligned}$$

whence

$$\begin{aligned}
\frac{2T(\theta+M\gamma(1-f))^3}{\gamma(1-f)}\beta' &= (4\alpha'(1-f)-1)(\theta+M\gamma(1-f)) - (1-f)(4\alpha'\theta+4\beta' M\gamma(1-f)-M\gamma) \\
\frac{2T(\theta+M\gamma(1-f))^3}{\gamma(1-f)}\beta' &= -\theta+4M\alpha'\gamma(1-f)^2-4\beta' M\gamma(1-f)^2 \\
\theta\gamma(1-f) &= \left[4M\gamma^2(1-f)^3\right]\alpha' - \left[4M\gamma^2(1-f)^3+2T(\theta+M\gamma(1-f))^3\right]\beta'
\end{aligned}$$

Flipping signs, we can write the two expressions

$$\begin{aligned}
\left[4M^2\gamma^2(1-f)^2+2t(\theta+M\gamma(1-f))^3\right]\alpha' + \left[-4M^2\gamma^2(1-f)^2\right]\beta' &= -M^2\gamma^2(1-f) \\
\left[-4M\gamma^2(1-f)^3\right]\alpha' + \left[4M\gamma^2(1-f)^3+2T(\theta+M\gamma(1-f))^3\right]\beta' &= -\theta\gamma(1-f)
\end{aligned}$$

as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}$$

where

$$
\begin{aligned}
A &= 4M^2\gamma^2(1-f)^2 + 2t(\theta + M\gamma(1-f))^3, & B &= -4M^2\gamma^2(1-f)^2 \\
C &= -4M\gamma^2(1-f)^3, & D &= 4M\gamma^2(1-f)^3 + 2T(\theta + M\gamma(1-f))^3 \\
E &= -M^2\gamma^2(1-f), & F &= -\theta\gamma(1-f)
\end{aligned}
$$

Letting $z = (\theta + M\gamma(1-f))$, note that

$$
\begin{aligned}
\left|\left[\begin{array}{cc} A & B \\ C & D \end{array}\right]\right| &= \left(4M^2\gamma^2(1-f)^2 + 2tz^3\right)\left(4M\gamma^2(1-f)^3 + 2Tz^3\right) - \left(-4M^2\gamma^2(1-f)^2\right)\left(-4M\gamma^2(1-f)^3\right) \\
&= 16M^3\gamma^4(1-f)^5 + 4M^2\gamma^2(1-f)^2 2Tz^3 + 4M\gamma^2(1-f)^3 2tz^3 + 4tTz^6 - 16M^3\gamma^4(1-f)^5 \\
&= 8M^2T\gamma^2(1-f)^2z^3 + 8Mt\gamma^2(1-f)^3z^3 + 4tTz^6 \qquad > \qquad 0
\end{aligned}
$$

so $\left[\begin{array}{cc} A & B \\ C & D \end{array}\right]$ is invertible, and we therefore have

$$
\left[\begin{array}{c} \alpha' \\ \beta' \end{array}\right] = \left[\begin{array}{cc} A & B \\ C & D \end{array}\right]^{-1}\left[\begin{array}{c} E \\ F \end{array}\right]
$$

Going back to our FOC, and setting $f_1 = f_2 = f$ and $\alpha = \beta = \frac{1}{2}$, we have

$$
\begin{aligned}
0 &= \frac{1}{f} - \frac{1}{1-f} + 2\beta' + 4\alpha' - 2\frac{2\alpha'\theta + 2\beta' M\gamma(1-f) - M\gamma}{\theta + M\gamma(1-f)} \\
&= \frac{1}{f} - \frac{1}{1-f} + 4\alpha' - \frac{4\alpha'\theta}{\theta + M\gamma(1-f)} + 2\beta' - \frac{4\beta' M\gamma(1-f)}{\theta + M\gamma(1-f)} + \frac{2M\gamma}{\theta + M\gamma(1-f)} \\
&= \frac{1}{f} - \frac{1}{1-f} + \frac{4M\gamma(1-f)}{\theta + M\gamma(1-f)}\alpha' + \frac{2\theta - 2M\gamma(1-f)}{\theta + M\gamma(1-f)}\beta' + \frac{2M\gamma}{\theta + M\gamma(1-f)} \\
&= \frac{1}{f} - \frac{1}{1-f} + \left[\begin{array}{cc} \frac{4M\gamma(1-f)}{\theta + M\gamma(1-f)} & \frac{2\theta - 2M\gamma(1-f)}{\theta + M\gamma(1-f)} \end{array}\right]\left[\begin{array}{c} \alpha' \\ \beta' \end{array}\right] + \frac{2M\gamma}{\theta + M\gamma(1-f)} \\
&= \frac{1}{f} - \frac{1}{1-f} + \left[\begin{array}{cc} \frac{4M\gamma(1-f)}{\theta + M\gamma(1-f)} & \frac{2\theta - 2M\gamma(1-f)}{\theta + M\gamma(1-f)} \end{array}\right]\left[\begin{array}{cc} A & B \\ C & D \end{array}\right]^{-1}\left[\begin{array}{c} E \\ F \end{array}\right] + \frac{2M\gamma}{\theta + M\gamma(1-f)}
\end{aligned}
$$

concluding the proof. □

S.4 Proof of Proposition 4

If the two platforms colluded by setting identical fee levels $f_1 = f_2 = f$, this would lead to equal market shares $\alpha = \beta = \frac{1}{2}$ and a buyer price

$$
P^D = \frac{N_i}{N_i\theta + M_i\gamma(1-f_i)} = \frac{1}{\theta + M\gamma(1-f)}
$$

at each platform. Demand at each platform would be

$$Q_i = M_i \gamma P_i^S = \frac{1}{2} M\gamma \frac{1-f}{\theta + M\gamma(1-f)}$$

giving each platform revenue of

$$\Pi_i = Q_i f_i P_i^D = \frac{1}{2} \frac{M\gamma f(1-f)}{(\theta + M\gamma(1-f))^2}$$

Taking the log and dropping the additive constants, maximizing

$$\log f + \log(1-f) - 2\log(\theta + M\gamma(1-f))$$

gives the first-order condition

$$\begin{aligned}
0 &= \frac{1}{f} - \frac{1}{1-f} + \frac{2M\gamma}{\theta + M\gamma(1-f)} \\
0 &= (1-f)(\theta + M\gamma(1-f)) - f(\theta + M\gamma(1-f)) + 2M\gamma f(1-f) \\
0 &= \theta(1-2f) + M\gamma(1-f)(1-2f) + 2M\gamma f - 2M\gamma f^2 \\
0 &= \theta - 2\theta f + M\gamma - 3M\gamma f + 2M\gamma f^2 + 2M\gamma f - 2M\gamma f^2 \\
0 &= \theta - 2\theta f + M\gamma - M\gamma f \\
f &= \frac{\theta + M\gamma}{2\theta + M\gamma}
\end{aligned}$$

Along with equal market shares, this implies a buyer price

$$P^D = \frac{1}{\theta + M\gamma\left(1 - \frac{\theta + M\gamma}{2\theta + M\gamma}\right)} = \frac{2\theta + M\gamma}{\theta(2\theta + M\gamma) + M\gamma\left(2\theta + M\gamma - (\theta + M\gamma)\right)} = \frac{2\theta + \gamma M}{2\theta(\theta + \gamma M)}$$

and seller price

$$P^S = (1-f)P^D = \left(1 - \frac{\theta + M\gamma}{2\theta + M\gamma}\right)\frac{2\theta + \gamma M}{2\theta(\theta + \gamma M)} = \frac{1}{2(\theta + \gamma M)}$$

which are the same as the collusive outcome in the price-setting game, and therefore lead to the same level of profits. □